2.2 PREVIOUS WORK IN PARTICIPATING GENERATOR IDENTIFICATION

In the study of IAOs, a phenomenon resulting from generators exchanging energy in an oscillatory manner, the question naturally arises: Which generators are most heavily involved in the energy exchange ? The answer to this question is important because it can lead to mitigation of the oscillation via control of these generators. Three of the most effective methods of participating generator identification are briefly outlined in this section. It is assumed that the frequency of the inter-area mode is known.

A linearized N-generator model of the power system is given by

$$\dot{\Delta x} = A\Delta x \tag{2.19}$$

where Δx is the state vector which includes, at a minimum, the rotor angles and speeds of all generators. The same problem formulation may be used when the state vector includes the states of the generator controls. The eigenvalues of this model are defined as those values of λ that satisfy

$$det[\lambda I - A] = 0.$$

For each eigenvalue λ_i there exists an N-element vector p_i, called the right eigenvector of A, such that

$$Ap_i = \lambda_i p_i,$$

and an N-element vector q_i, called the left eigenvector of A, such that

$$q_i^T A = \lambda_i q_i^T.$$

Additionally, the matricies P and Q are composed of all right and left eigenvectors, respectively [17], i.e.,

$$P = [p_1 ... p_N], \quad Q^T = \begin{bmatrix} q_1^T \\ \vdots \\ q_N^T \end{bmatrix}$$

Assuming all eigenvalues are distinct, consider two eigenvalues λ_i and λ_j. Then

- for $i \neq j$, q_i and p_j must be orthogonal: $q_i^T p_j = 0$, and
- for $i = j$, $q_i^T p_j = k_i$, and it is assumed q_i and p_j can be made orthonormal so that $q_i^T p_j = 1$ [42]. It is assumed that this equation holds for the remainder of this section.

These two relations imply that $Q^T = P^{-1}$, i.e., P and Q are orthonormal matricies.

Assuming the vector x has initial conditions $x(0)$, the system will 'ring down' according to

$$x(s) = [sI - A]^{-1} x(0). \tag{2.20}$$

where the notation Δx has been abbreviated by x. Noting that $PP^{-1} = I$ and $[Q^T]^{-1} Q^T = I$, equation 2.20 may be rewritten as

$$x(s) = PP^{-1}[sI - A]^{-1}[Q^T]^{-1} Q^T x(0)$$

$$x(s) = P \left\{ P^{-1}[sI - A]^{-1}[Q^T]^{-1} \right\} Q^T x(0) \tag{2.21}$$

The expression inside the curly brackets of equation 2.21 can be expressed as [1]

$$P^{-1}[sI - A]^{-1}[Q^T]^{-1} = [sI - \Lambda]^{-1} \tag{2.22}$$

where

$$\Lambda = diag(\lambda_i)$$

Therefore equation 2.21 becomes

$$x(s) = P[sI - \Lambda]^{-1}Q^T x(0) \tag{2.23}$$

Equation 2.23 may be expanded as

$$x(s) = \sum_{i=1}^{N} \frac{p_i(q_i^T x(0))}{s - \lambda_i} = \sum_{i=1}^{N} \frac{(q_i^T x(0))p_i}{s - \lambda_i} \tag{2.24}$$

Transforming equation 2.24 to the time domain,

$$x(t) = \sum_{i=1}^{N}[(q_i^T x(0))e^{\lambda_i t}]p_i \tag{2.25}$$

[1] Since A is square and assumed to have distinct eigenvalues, it has full rank. Given that P is the matrix of N independent right eigenvectors, A may be diagnalized by

$$P^{-1}AP = \Lambda = diag(\lambda_i)$$

Since the right and left eigenvector matricies P and Q^T are orthogonal, $P = [Q^T]^{-1}$, and

$$P^{-1}A[Q^T]^{-1} = \Lambda = diag(\lambda_i)$$

If P and Q^T are the right and left eigenvector matricies for A, then they are also the right and left eigenvector matricies for $[sI - A]$. Therefore

$$P^{-1}[sI - A][Q^T]^{-1} = [sI - \Lambda]$$

Recalling $[BCD]^{-1} = D^{-1}C^{-1}B^{-1}$, both sides of the previous equation may be inverted to yield

$$Q^T[sI - A]^{-1}[P] = [sI - \Lambda]^{-1}$$

Again using the orthogonality condition $P = [Q^T]^{-1}$,

$$P^{-1}[sI - A]^{-1}[Q^T]^{-1} = [sI - \Lambda]^{-1}$$

2.2.1 The Eigenvector Inspection Method

The eigenvector inspection method was developed by de Mello, et. al., [6] and has been used often in the industry [31, 22]. It relies on the fact that the right eigenvector p_i corresponding to λ_i gives information about the *shape* of the i-th mode. The idea of mode shape can best be grasped by inspecting equation 2.25 and noting that p_i determines the distribution of the mode through the state variables. (Note that $q_i^T x(0) e^{\lambda_i t}$ is a scalar and p_i and $x(t)$ are vectors.) If the states are limited to only those corresponding to the rotor angles and speeds of the generators, then each element of the right eigenvector gives the distribution of the mode in a particular generator's rotor angle or speed. Therefore inspection of the elements of the right eigenvector associated with λ_i gives an indication of which generators are most actively swinging against each other in this mode; those generators corresponding to elements having large positive values swing against those generators corresponding to elements having large negative values.

2.2.2 The Participation Factor Method

The participation factor method depends on both the right and left eigenvectors to compute a participation factor for the k-th state in the i-th mode. A derivation for the participation factor follows (adapted from [29]). Crucial to the derivation is the intuitive understanding of the left eigenvector: q_i determines the influence of the initial conditions $x(0)$ on the mode λ_i. In effect, the left eigenvector together with the initial conditions determines the magnitude of the mode. Again, this idea is most easily grasped by inspection of equation 2.25.

To begin the derivation, a new state variable ξ_i is defined such that

$$\xi_i = q_i^T x \tag{2.26}$$

$$\Rightarrow \dot{\xi}_i = q_i^T \dot{x}$$

where the vector Δx is abbreviated as x as before. Multiplying both sides of equation 2.19 by q_i^T gives

$$q_i^T \dot{x} = q_i^T A x \tag{2.27}$$

By the definition of the left eigenvector, $q_i^T A = \lambda_i q_i^T$, we substitute for the right side of equation 2.27 to get

$$q_i^T \dot{x} = \lambda_i q_i^T x \tag{2.28}$$

Substituting ξ_i and $\dot{\xi}_i$ from 2.26 into equation 2.28, we have

$$\dot{\xi}_i = \lambda_i \xi_i \tag{2.29}$$

which shows that the transformed state variable ξ_i is only associated with the i-th mode and no other modes. Therefore the state variables that are significant in ξ are the same state variables that are significant in the i-th mode.

Assuming only the i-th mode is excited, ξ_i can be written in terms of all the state variables by substitution of equation 2.25 into equation 2.26:

$$\xi_i = q_i^T x = q_i^T \sum_{j=1}^{N} \left[(q_j^T x(0)) e^{\lambda_j t} \right] p_j = \sum_{j=1}^{N} \left[(q_i^T x(0)) e^{\lambda_i t} \right] q_i^T p_j \tag{2.30}$$

Recalling the orthogonality property $q_i^T p_j = 0$ for $i \neq j$, equation 2.30 simplifies to

$$\xi_i = \left(q_i^T x(0) \right) q_i^T p_i e^{\lambda_i t} = \left(q_i^T x(0) \right) \left[\sum_{k=1}^{N} q_{ki} p_{ki} \right] e^{\lambda_i t} \tag{2.31}$$

Finally,

$$\xi_i = (q_i^T x(0)) [\sum_{k=1}^{N} \rho_{ki}] e^{\lambda_i t} \tag{2.32}$$

where

$$\rho_{ki} = q_{ki} p_{ki}$$

is defined as the participation factor of the k-th state in the i-th mode. Also, since $q_i^T p_i = 1$, it must be true that

$$\sum_{k=1}^{N} \rho_{ki} = 1$$

Generators corresponding to the largest participation factors for the i-th mode will most strongly affect this mode.

2.2.3 The Residue Method

The residue method provides for calculation of eigenvalue sensitivities with respect to a specific system parameter. The *residue* r_i corresponding to an eigenvalue λ_i is the magnitude of the initial size of the transient characterized by the term $r_i \exp(-\lambda_i t)$ [42]. The power system is represented in the form:

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

where u and y are the input and output variables, respectively, for a specific generator j, and it is assumed that λ_i is an eigenvalue of the system. Adding PSS to generator j having gain k_j and transfer function $f_j(s, k_j)$ is equivalent to adding an additional feedback from y and mixing it with u. It can be shown that the sensitivity of λ_i with respect to k_j is:

$$\lambda_i' = r_i f_j'(\lambda_i, k_j)$$

where the residues are calculated as:

$$r_i = C^T p_i q_i^T B.$$

Those generators with the largest λ_i' participate most strongly in the mode. It is interesting to note that the participation factor method is a specific case of the residue method when the PSS has no lead or lag, i.e. when the PSS is a pure gain.