EE 455, HW2, Spring 2005, Dr. McCalley
Due Monday, February 7, 2005

SOLUTIONS

1. (a) Given I_a=1/_60º, I_b=1/_-60º, I_c=0, find the sequence components (the 012 components corresponding to the a-phase currents).
   (a) Find the 012 components corresponding to the c-phase.
   (b) Sum the 012 components corresponding to the c-phase to see if they sum to 0.

Part a)

$$\begin{aligned}
I_1 &= \frac{1}{3}\left(I_a + aI_b + a^2 I_c\right) \\
&= \frac{1}{3}\left(1\angle 60° + 1\angle 60° + 0\right) \\
&= 0.667\angle 60° \\
I_2 &= \frac{1}{3}\left(I_a + a^2 I_b + aI_c\right) \\
&= \frac{1}{3}\left(1\angle 60° + 1\angle 180° + 0\right) \\
&= 0.333\angle 120° \\
I_0 &= \frac{1}{3}\left(I_a + I_b + I_c\right) \\
&= \frac{1}{3}\left(1\angle 60° + 1\angle -60° + 0\right) \\
&= 0.333\angle 0°
\end{aligned}$$

Part b)

$$\begin{aligned}
I_{c1} &= aI_1 = 0.667\angle 180° \\
I_{c2} &= a^2 I_2 = 0.333\angle 360° \\
I_{c0} &= I_0 = 0.333\angle 0°
\end{aligned}$$

Part c)

$$\begin{aligned}
I_c &= I_{c1} + I_{c2} + I_{c0} \\
&= -0.667 + 0.333 + 0.333 \cong 0
\end{aligned}$$

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2. A Y-connected solidly grounded voltage source with unbalanced voltage is applied to a balanced line and load. Assume that there is no mutual coupling between phases. The load is connected delta. Data for the problem is as follows:
   - Voltage source: V_{ag}=277/_0º, V_{bg}=260/_-120º, V_{cg}=295/_115º
   - Load impedance : 30/_40º.
   - Line impedance: 1/_85º.

   (a) Compute sequence source voltages.
   (b) Compute the 012 currents in the line.
   (c) Compute the abc currents in the line.

Note, when converting the delta load to wye, it must remain **ungrounded**.

Part a)

$$\begin{aligned} V_1 &= \frac{1}{3}\left(V_{ag} + aV_{bg} + a^2V_{cg}\right) \\ &= \frac{1}{3}\left(277\angle 0° + 260\angle 0° + 295\angle -5°\right) \\ &= 277.1\angle -1.772° \quad \text{volts} \end{aligned}$$

$$\begin{aligned} V_2 &= \frac{1}{3}\left(V_{ag} + a^2V_{bg} + aV_{cg}\right) \\ &= \frac{1}{3}\left(277\angle 0° + 260\angle 120° + 295\angle 235°\right) \\ &= 9.218\angle 216.59° \quad \text{volts} \end{aligned}$$

$$\begin{aligned} V_0 &= \frac{1}{3}\left(V_{ag} + V_{bg} + V_{cg}\right) \\ &= \frac{1}{3}\left(277\angle 0° + 260\angle -120° + 295\angle 115°\right) \\ &= 15.912\angle 62.11° \quad \text{volts} \end{aligned}$$

Part b)

$$I_1 = \frac{V_1}{Z_{L1} + \frac{Z_\Delta}{3}} = \frac{277.1\angle -1.722°}{10.73\angle 43.78°} = 25.82\angle -45.55° \text{ A}$$

$$I_2 = \frac{V_2}{Z_{L2} + \frac{Z_\Delta}{3}} = \frac{9.218\angle 216.59°}{10.73\angle 43.78°} = 0.8591\angle 172.81° \text{ A}$$

$$I_0 = 0 \text{ A}$$

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Part c)

$$
\begin{aligned}
I_a &= 0 + 25.82\angle -45.55° + 0.8591\angle 172.81° \\
&= 25.15\angle -46.76° \quad \text{A} \\
I_b &= 0 + (25.82\angle -45.55°)(1\angle 240°) + (0.8591\angle 172.81°)(1\angle 120°) \\
&= 25.71\angle 196.34° \quad \text{A} \\
I_c &= 0 + (25.82\angle -45.55°)(1\angle 120°) + (0.8591\angle 172.81°)(1\angle 240°) \\
&= 26.62\angle 73.77° \quad \text{A}
\end{aligned}
$$

3. A 3-phase transmission line has series reactance of X_S in all three phases and mutual inductance between every pair of phases of X_m. Voltages at one end of the line are V_a, V_b, V_c, and voltages at the other end of the line are V'_a, V'_b, and V'_c. Unbalanced currents I_a, I_b, and I_c flow in the phases.
   (a) Write a matrix equation $\underline{V}_{abc}-\underline{V}'_{abc}=\underline{Z}\,\underline{I}_{abc}$ and substitute 012 quantities for V_{abc}, V'_{abc}, and I_{abc}.
   (b) Determine the impedance matrix that relates the 012 voltages to the 012 currents.
   (c) Are the three sequence circuits coupled?

Part a)

In abc: $\left[V_{abc}\right]-\left[V'_{abc}\right]=\left[Z_{abc}\right]\left[I_{abc}\right]$

In 012: $\left[V_{012}\right]-\left[V'_{012}\right]=\left[Z_{012}\right]\left[I_{012}\right]$

Part b)

$$
\begin{aligned}
\left[Z_{012}\right] &= \left[A\right]^{-1}\left[Z_{abc}\right]\left[A\right] \\
&= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} j \begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \\
&= \frac{1}{3} j \begin{bmatrix} X_s + 2X_m & X_s + 2X_m & X_s + 2X_m \\ X_s - X_m & aX_s + (1+a^2)X_m & a^2X_s + (1+a)X_m \\ X_s - X_m & a^2X_s + (1+a)X_m & aX_s + (1+a^2)X_m \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \\
&= j \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - 2X_m & 0 \\ 0 & 0 & X_s - 2X_m \end{bmatrix}
\end{aligned}
$$

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Part c)

No, the mutual coupling has been eliminated.

4. A 3-phase transmission line has series reactance of X_S in all three phases and mutual inductances between phases of X_{ab}, X_{bc}, and X_{ca}. Everything else is the same as in problem 3. Repeat (a), (b), and (c) of problem 3.

Part a)

In abc: $[V_{abc}]-[V'_{abc}]=[Z_{abc}][I_{abc}]$

In 012: $[V_{012}]-[V'_{012}]=[Z_{012}][I_{012}]$

Part b)

$$[Z_{012}] = [A]^{-1}[Z_{abc}][A]$$

$$= \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & a & a^2\\ 1 & a^2 & a\end{bmatrix} j\begin{bmatrix}X_s & X_{ab} & X_{ca}\\ X_{ab} & X_s & X_{bc}\\ X_{ca} & X_{bc} & X_s\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{bmatrix}$$

$$= \frac{1}{3} j\begin{bmatrix}X_s+X_{ab}+X_{ca} & X_{ab}+X_s+X_{bc} & X_{ca}+X_{bc}+X_s\\ X_s+aX_{ab}+a^2X_{ca} & X_{ab}+aX_s+a^2X_{bc} & X_{ca}+aX_{bc}+a^2X_s\\ X_s+a^2X_{ab}+aX_{ca} & X_{ab}+a^2X_s+aX_{bc} & X_{ca}+a^2X_{bc}+aX_s\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{bmatrix}$$

$$= \frac{1}{3} j\begin{bmatrix}3X_s+2(X_{ab}+X_{bc}+X_{ca}) & (1+a^2)X_{ab}-X_{bc}+(1+a)X_{ca} & (1+a)X_{ab}-X_{bc}+(1+a^2)X_{ca}\\ (1+a)X_{ab}-X_{bc}+(1+a^2)X_{ca} & 3X_s-X_{ab}-X_{bc}-X_{ca} & 2(aX_{ab}+X_{bc}+a^2X_{ca})\\ (1+a^2)X_{ab}-X_{bc}+(1+a)X_{ca} & 2(a^2X_{ab}+X_{bc}+aX_{ca}) & 3X_s-X_{ab}-X_{bc}-X_{ca}\end{bmatrix}$$

Part c)

Yes, the mutual coupling still exist between the phases (off-diagonal terms).

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5. As you know, the power delivered to a three-phase network is given by

$$S = V_{ag}I_a^* + V_{bg}I_b^* + V_{cg}I_c^* = \underline{V}_{abc}^T \underline{I}_{abc}^*$$

Using the above relation, develop an expression for the power delivered to the three phase network in terms of only sequence quantities.

In matrix notation:

$$S_{3\phi} = [V_{abc}]^T [I_{abc}]^*$$

$$= \{[A][V_{012}]\}^T \{[A][I_{012}]\}^*$$

$$= [V_{012}]^T [A]^T [A]^* [I_{012}]^*$$

Now

$$[A]^T [A]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$S_{3\phi} = 3[V_{012}]^T [I_{012}]^*$$

$$= 3\left[V_0 I_0^* + V_1 I_1^* + V_2 I_2^*\right]$$