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# Transmission Investment

Oscar Volij

Iowa State University

# Motivation

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There are two regions: Oblivia and Rodrigombia. Each of the two regions has generators and consumers.

# The main actors

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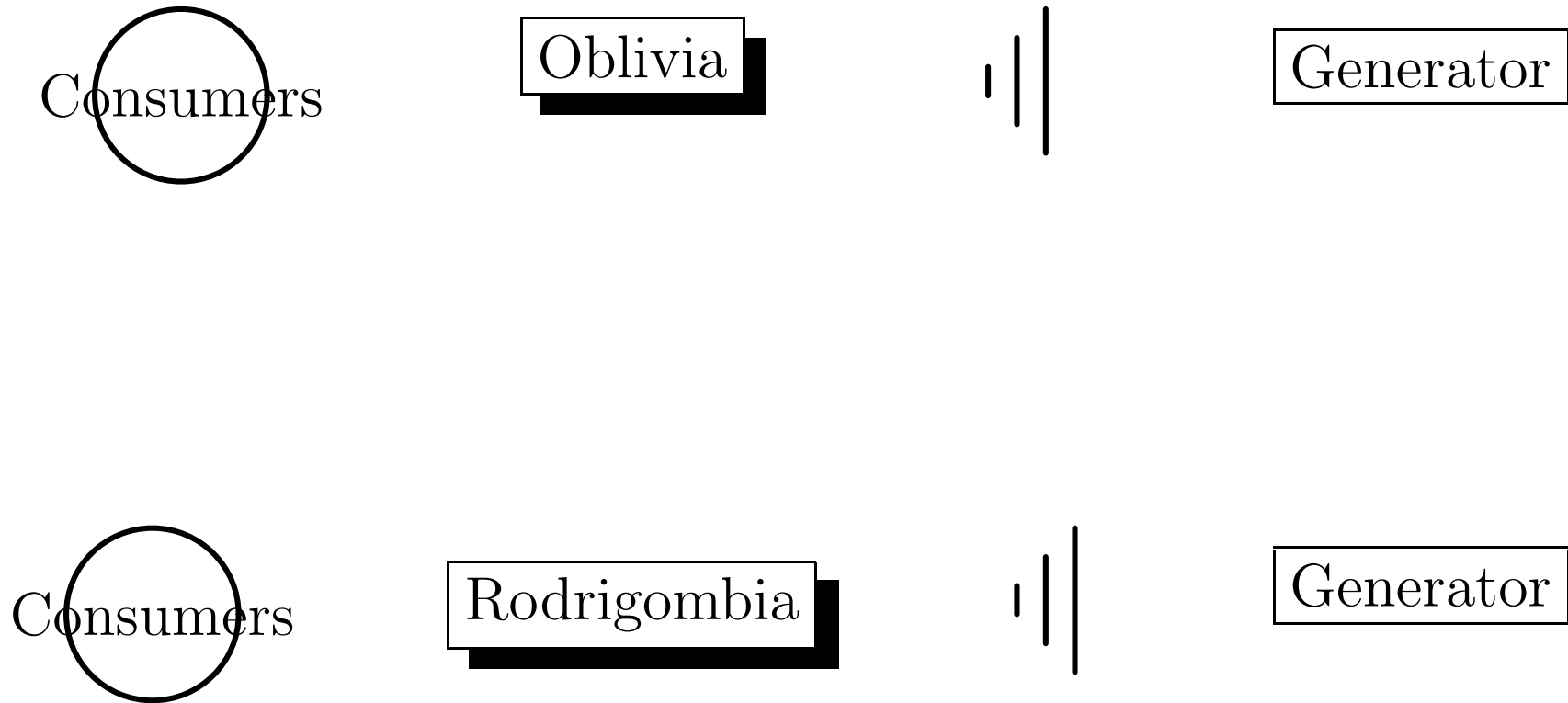


Figure 1: The main actors

# Oblivia

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The generator in Oblivia is characterized by the following cost function:

$$C_O(q) = 4q + \frac{q^2}{2}.$$

The consumers in Oblivia have the following utility function:

$$U_O(m, x) = m + \frac{(100 - x)x}{2}$$

# Rodrigombia

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The generator in Rodrigombia is characterized by the following cost function:

$$C_R(q) = 40q + \frac{q^2}{2}.$$

The consumers in Rodrigombia have the following utility function:

$$U_R(m, x) = m + \frac{(120 - x)x}{2}$$

# The Generators Problem

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Generators take the price of power  $p$  as given, and generate the amount of power that brings their profits to the maximum.

Formally, node  $i$ 's generator solves

$$\max_{q \geq 0} pq - C_i(q).$$

The solution to this problem is the quantity that solves the first order condition

$$p = \frac{\partial C_i}{\partial q}(q^*).$$

The solution to this equation is a function of the price, and is known as the *generator's supply function*.

# Oblivia's Supply Function

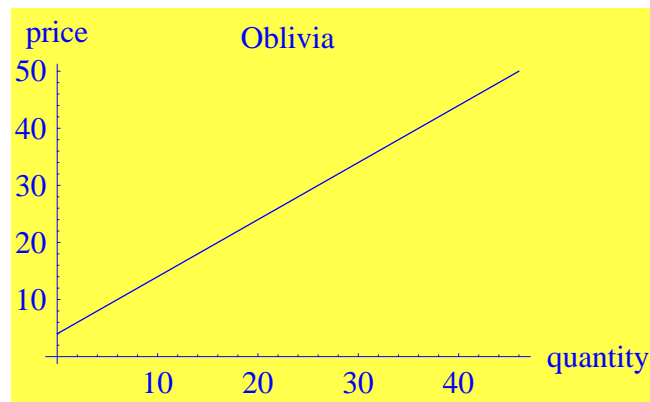
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In the case of Oblivia, the profit maximizing output solves

$$-4 + p - q = 0$$

Consequently, Oblivia's generator's supply function is

$$S_O(p) = -4 + p.$$



# Rodrigombia's Supply Function

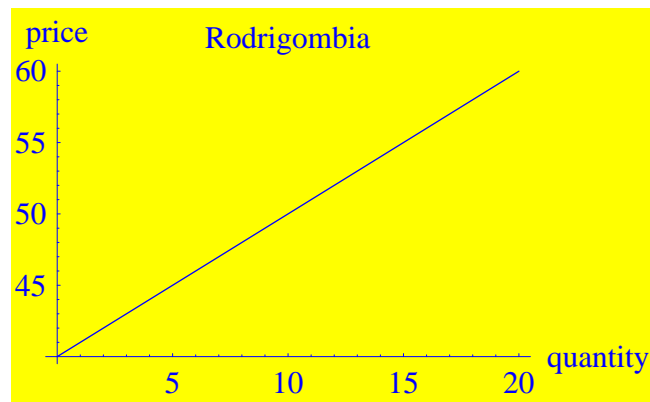
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In the case of Rodrigombia, the profit maximizing output solves

$$-40 + p - q = 0$$

Consequently, Rodrigombia's generator's supply function is

$$S_R(p) = -40 + p.$$



# The Consumers' Problem

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Consumers take the price of power  $p$  and their income  $I$  as given, and buy the amount of power that brings their utility to the maximum. Formally, node  $i$ 's consumer solves

$$\begin{aligned} \max_{m, q \geq 0} \quad & m + u_i(m, q) \\ \text{s.t.} \quad & m + pq = I. \end{aligned}$$

The solution to this problem is the quantity that solves the first order condition

$$p = \frac{\partial u_i}{\partial q}(q^*).$$

The solution to this equation is a function of the price, and is known as the *consumer's demand function*.

# Oblivia's Demand Function

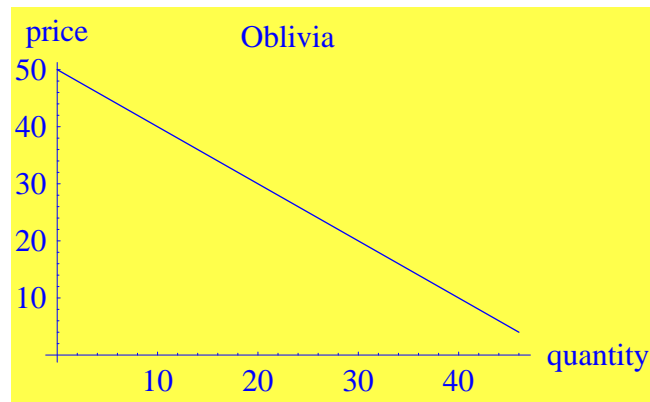
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In the case of Oblivia, the utility maximizing quantity solves

$$-p + \frac{100 - x}{2} - \frac{x}{2} = 0$$

Consequently, Oblivia's consumers' demand function is

$$D_O(p) = 50 - p.$$



# Rodrigombia's Demand Function

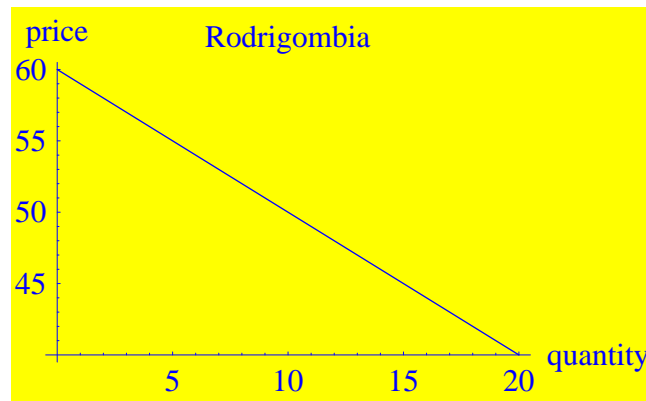
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In the case of Rodrigombia, the utility maximizing quantity solves

$$-p + \frac{120 - x}{2} - \frac{x}{2} = 0$$

Consequently, Rodrigombia's consumers' demand function is

$$D_R(p) = 60 - p.$$



# Transmission Firm

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There is a transmission investment firm that can build transmission capacity with the following cost function:

$$C_I(k) = \frac{15 k^2}{16}$$

# The Transmission Firm's Problem

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The transmission firm takes the price of transmission  $\mu$  as given, and build the transmission capacity that brings its profits to the maximum. Formally, the transmission investment firm solves

$$\max_{k \geq 0} \mu k - C_I(k).$$

The solution to this problem is the transmission capacity that solves the first order condition

$$\mu = \frac{\partial C_I}{\partial k}(k^*).$$

The solution to this equation is a function of the transmission price, and is known as the *transmission firm's supply function*.

# Transmission firm's Supply Function

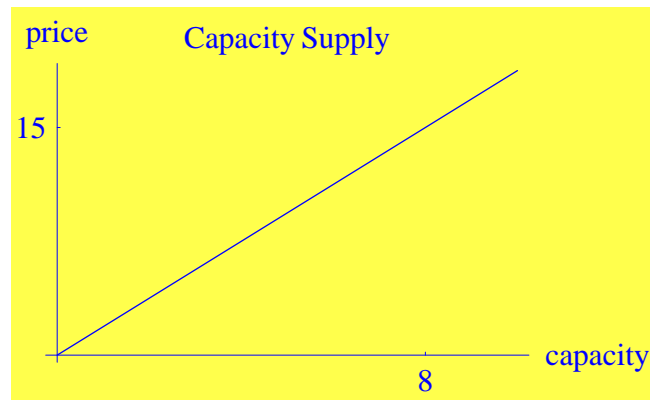
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In our numerical example, the profit maximizing capacity solves

$$\frac{-15k}{8} + \mu = 0$$

Consequently, the transmission firms's supply function is

$$S_I(\mu) = \frac{8\mu}{15}.$$



# Competitive equilibrium

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We are interested in finding A price  $p_O$  in Oblivia, a price  $p_R$  in Rodrigombia, and a transmission charge  $\tau$  such that

- $S_O(p_O) + S_R(p_R) = D_O(p_O) + D_R(p_R)$  (Demand equals supply)
- $S_I(\tau) = |S_O(p_O) - D_O(p_O)| = |D_R(p_R) - S_R(p_R)|$  (The power transmitted equals the capacity of the line that the transmission firm wants to build.)
- $|p_O - p_R| = \tau$  (One cannot make money by buying power from Oblivian generators, transmitting it through the line and selling it to Rodrigombian consumers.)
- $(p_O - p_R)(S_O(p_O) - D_O(p_O)) \leq 0$  (The flow of power goes from the inexpensive bus to the more expensive one.)

# Social Optimum

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- What is the socially optimal allocation of resources?
- What is the **consumption** and **generation** levels in each region, and **transmission capacity** that maximize the social surplus?

# Social optimum

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In order to find the social optimum we need to solve the following problem:

$$\begin{aligned} \max \quad & U_O(x_O) + U_R(x_R) - C_O(q_O) - C_R(q_R) - C_I(k) \\ \text{s.t.} \quad & \begin{cases} x_O + x_R = q_O + q_R \\ q_O - x_O \leq k \end{cases} \end{aligned}$$

# Social Optimum

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In our case, the problem is

$$\begin{aligned} \max \quad & \frac{(100-x_O)x_O}{2} + \frac{(120-x_R)x_R}{2} \\ & - \left(4q_O + \frac{q_O^2}{2}\right) - \left(40q_R + \frac{q_R^2}{2}\right) \\ & - \frac{15k^2}{16} \\ \text{s.t.} \quad & \begin{cases} x_O + x_R = q_O + q_R \\ q_O - x_O \leq k \end{cases} \end{aligned}$$

# Social optimum

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The Lagrangian is

$$\begin{aligned}\mathcal{L} = & \frac{(100 - x_O) x_O}{2} + \frac{(120 - x_R) x_R}{2} \\ & - 4q_O - \frac{q_O^2}{2} - 40q_R - \frac{q_R^2}{2} \\ & - \frac{15k^2}{16} \\ & - \lambda(x_O + x_R - q_O - q_R) \\ & - \mu(q_O - x_O - k)\end{aligned}$$

# Social Optimum

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The first order conditions are:

$$\frac{100 - x_O}{2} - \frac{x_O}{2} - \lambda + \mu = 0$$

$$\frac{120 - x_R}{2} - \frac{x_R}{2} - \lambda = 0$$

$$-4 - q_O + \lambda - \mu = 0$$

$$-40 - q_R + \lambda = 0$$

$$\frac{-15k}{8} + \mu = 0$$

$$q_O + q_R - x_O - x_R = 0$$

$$q_O - x_O - k = 0$$

The solution to this system of equations is

$$\{x_O \rightarrow 19, x_R \rightarrow 14, q_O \rightarrow 27, q_R \rightarrow 6, k \rightarrow 8, \lambda \rightarrow 46, \mu \rightarrow 15\}$$

# Competitive equilibrium

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**Question:** Can the above outcome be obtained as a result of decentralized trade?

**Answer:** Yes!

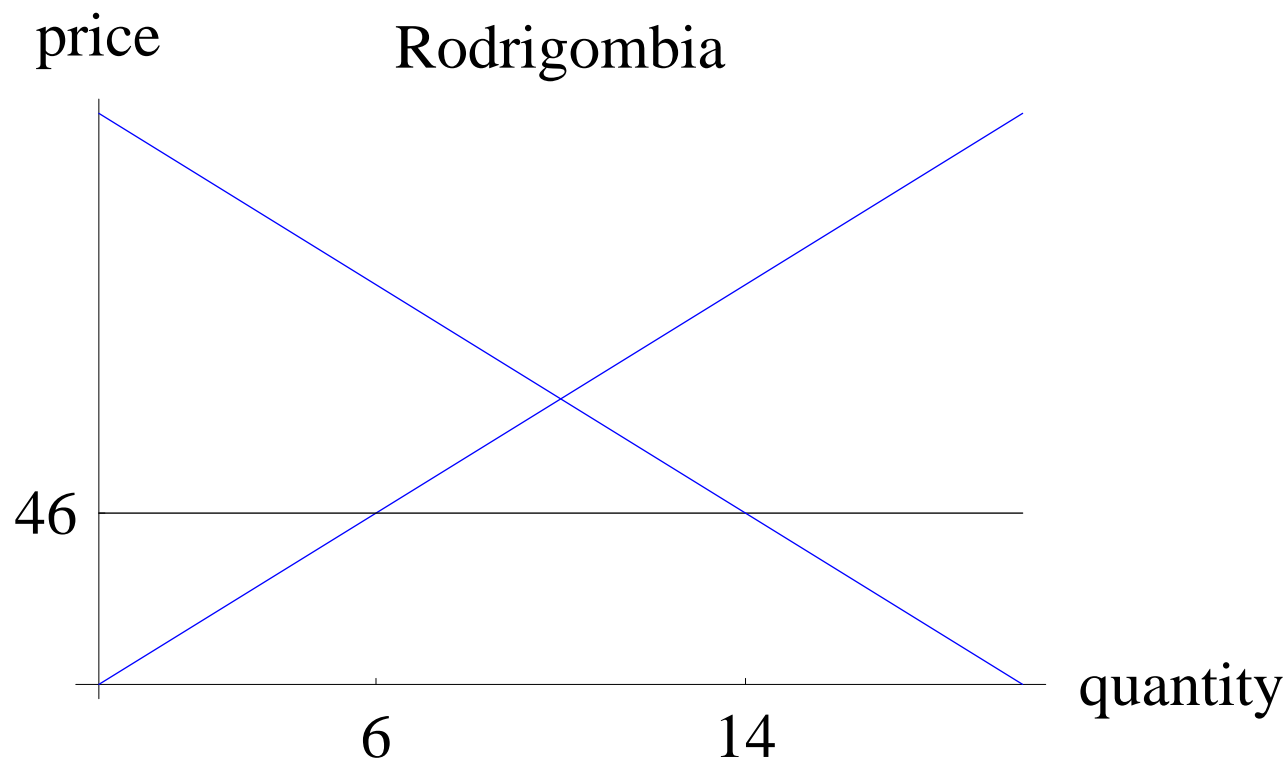
**Question:** How?

**Answer:** As follows.

# Competitive equilibrium

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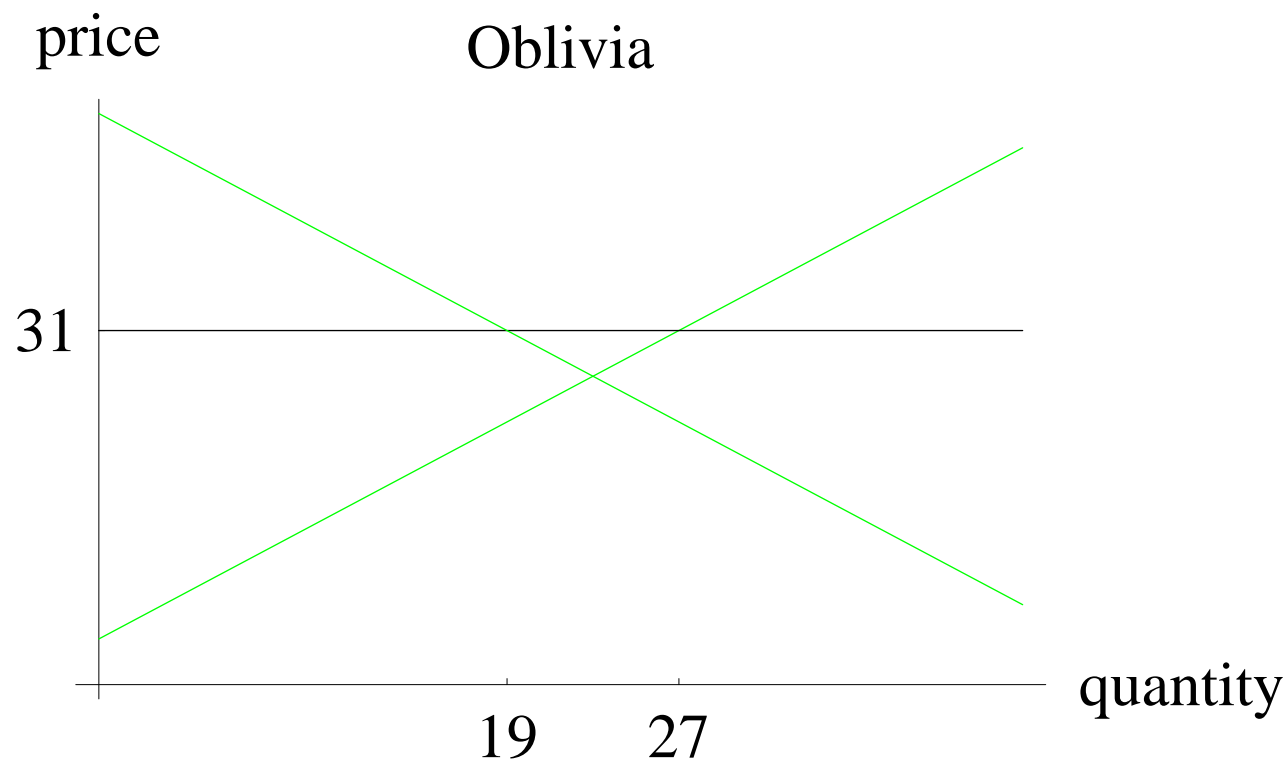
- What would happen in Rodrigombia if the local price of power was 46?
- To answer this question we need to look at the Rodrigombian market



# Competitive equilibrium

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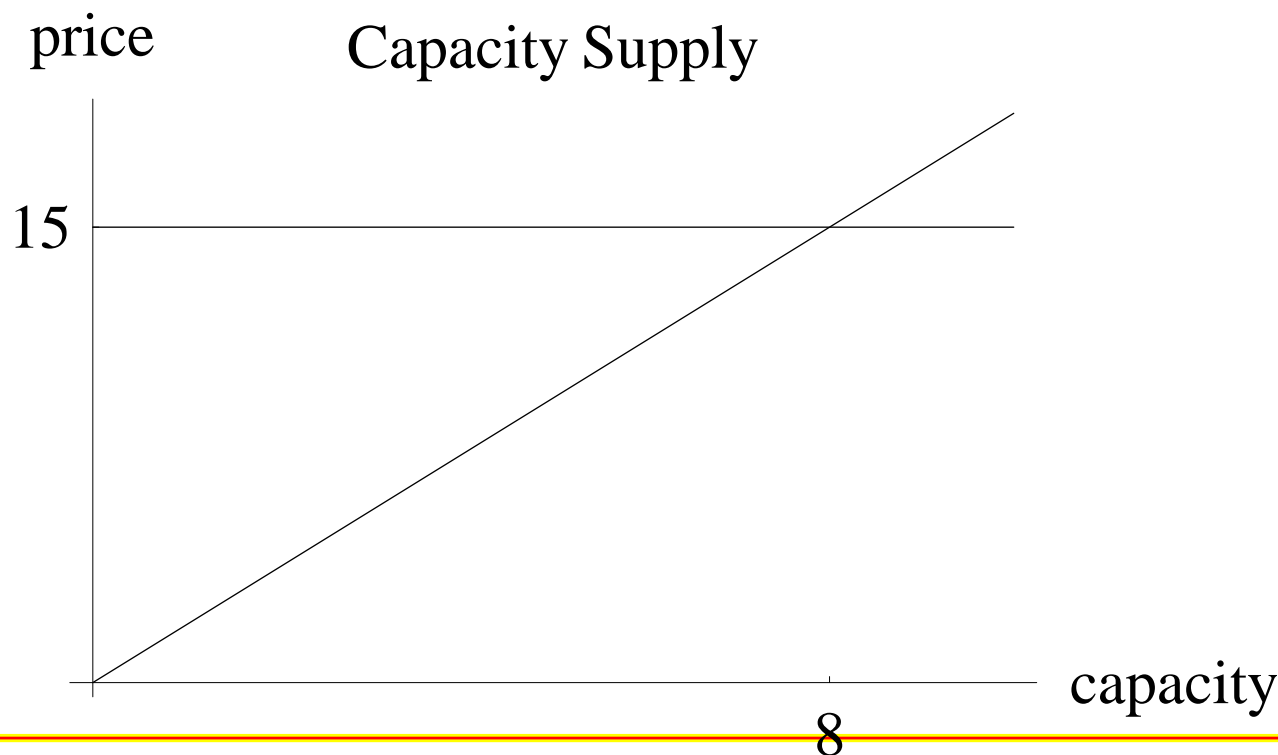
- What would happen in Oblivia if the local price of power was 31?
- To answer this question we need to look at the Oblivian market



# Competitive equilibrium

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- What would be the transmission capacity that the transmission company would be willing to build if the transmission price was 15?
- To answer this question we need to look at the transmission firm's supply function:



# Competitive equilibrium

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We see that if the price in Oblivia is 31 and the price in Rodrigombia is 46, the quantities demanded and supplied in each of the regions coincide with the socially optimal quantities, and that if the transmission price is 15 ( $= 46-31$ ) the transmission capacity built is also the socially optimal one.

# Competitive equilibrium

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In this equilibrium:

- Oblivian generators sell 27 units at \$31/MW
- Oblivian consumers buy 19 units at \$31/MW
- Rodrigombian generators sell 6 units at \$46/MW
- Rodrigombian consumers buy 14 units at \$46/MW

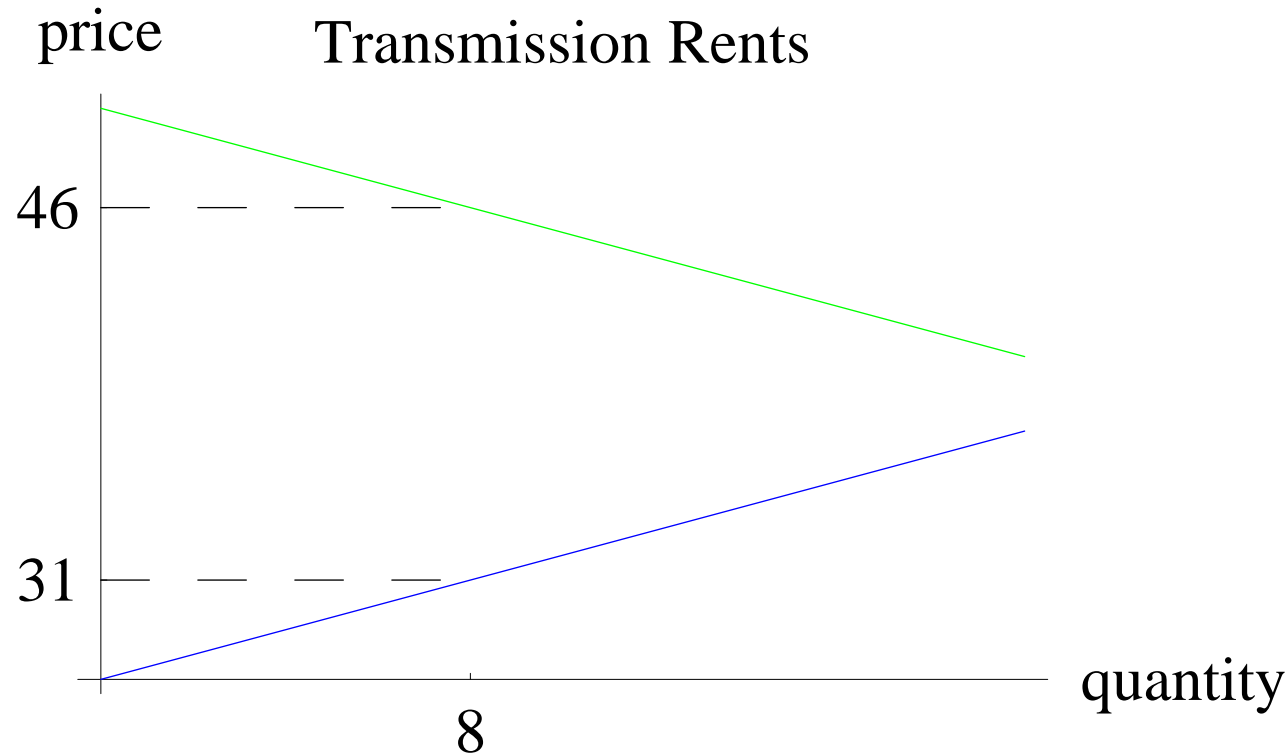
Therefore

- Oblivian generators export 8 units to Rodrigombian consumers
- Oblivian generators get  $\$8 \times 46$
- Rodrigombian consumers pay  $\$8 \times 31$
- Where does the difference go?

# Transmission rent

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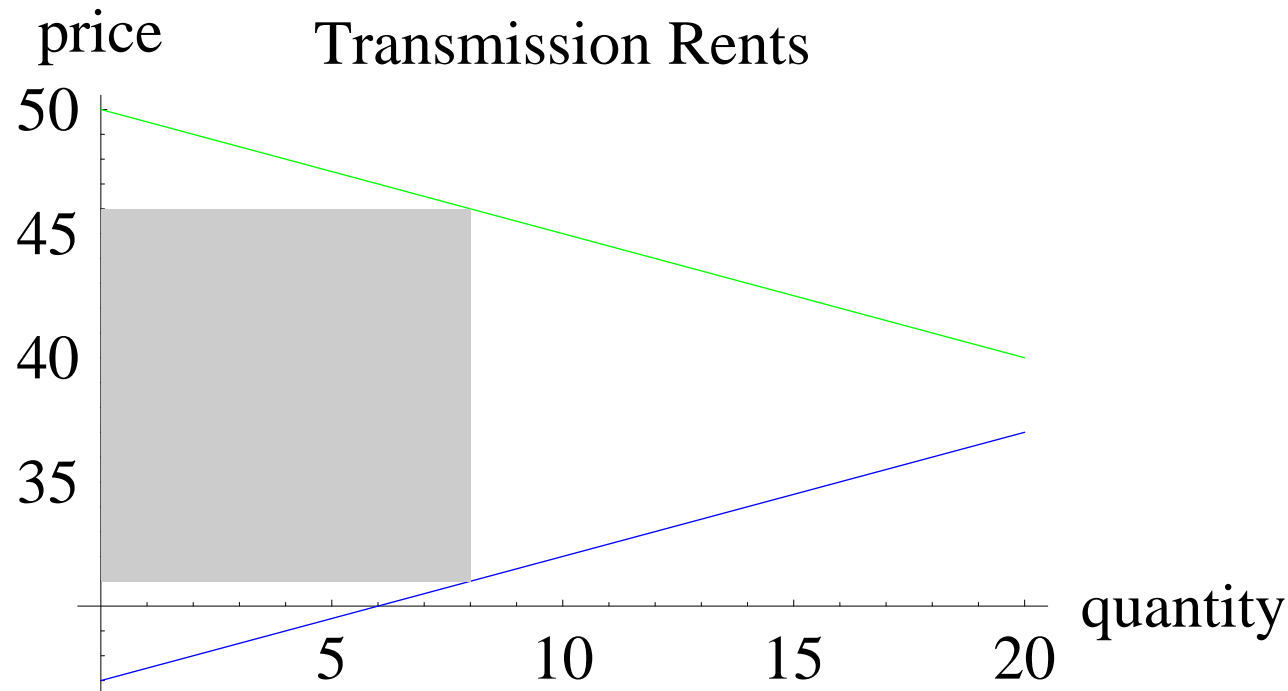
The difference goes to the transmission owners.



# Transmission rent

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The transmission owners charge \$4 for each unit that transits along the line.



# Competitive equilibrium

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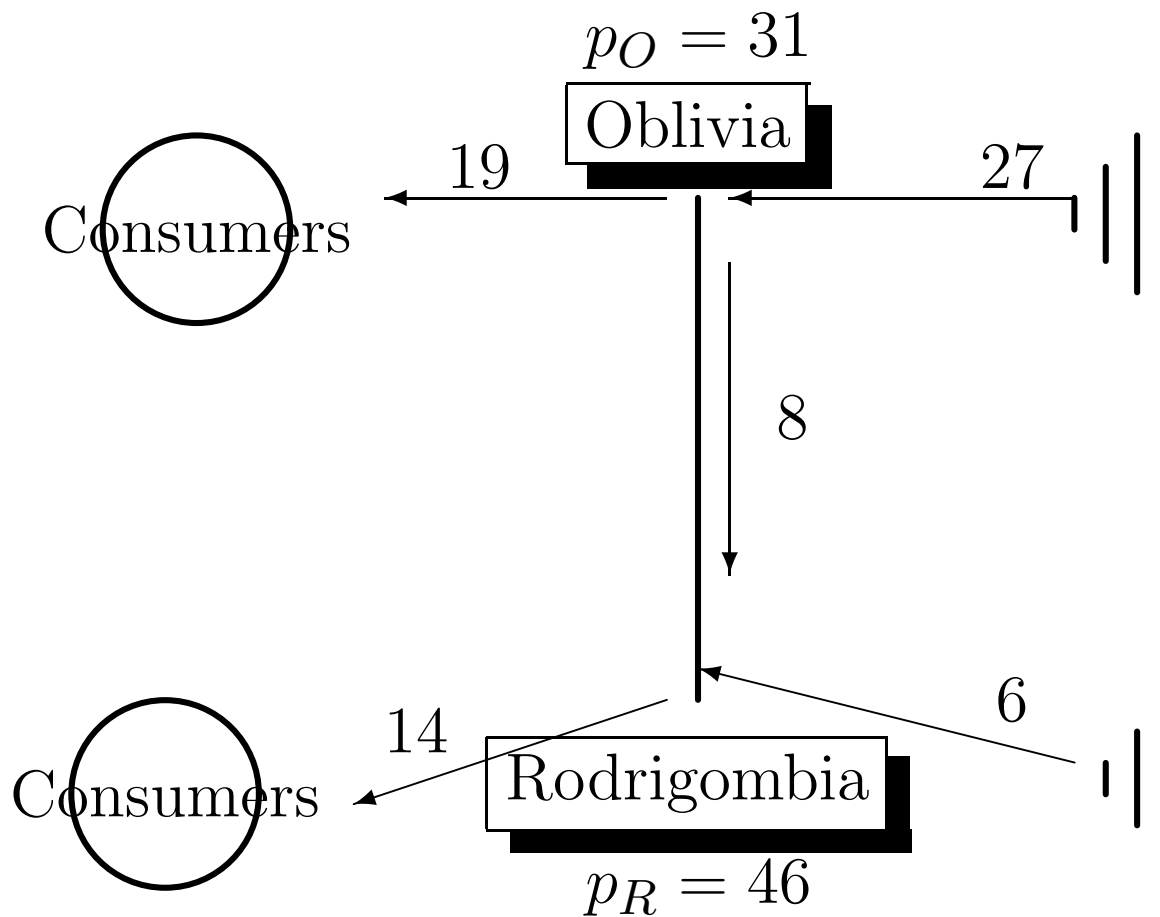


Figure 2: Equilibrium dispatch

# Competitive equilibrium

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How can we characterize our equilibrium?

The *competitive equilibrium* consists of

- A price of \$31/MW in Oblivia
- A price of \$46/MW in Rodrigombia
- A transmission charge of \$15/MW

such that

- $S_O(31) - D_O(31) = D_R(46) - S_R(46)$
- $S_I(15) = S_O(31) - D_O(46)$
- The power transmitted does not exceed the capacity of the line. In fact it equals the capacity of the line given that the transmission charge is positive
- One cannot make money by buying power from Oblivian generators, transmitting it through the line and selling it to Rodrigombian consumers.