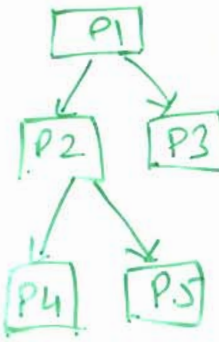
Branch & Bound Method for LMIP

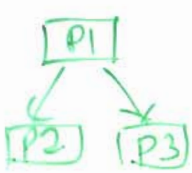


- Solve LMIP by solving a set of LP's
- The set of LP's to be solved arranged in a "tree" (tree = acyclic directed graph with one root node)
- Each node of tree corresponds to a **certain** LP. "Root" node corresponds to LP relaxation of given LMIP.
- After solving a LP, branch to two new LPs unless
  - (i) current LP infeasible, or
  - (ii) current LP provides a feasible solution for LMIP (in which case, a lower bound for LMIP is found)
  - (iii) current LP solution lower than current lower bound. (initial lower bound $= -\infty$)
- Terminate when no branching possible at any node.

- Example: $\max z = 8x_1 + 5x_2$

$$\text{s.t.} \quad x_1 + x_2 \leq 6$$
$$9x_1 + 5x_2 \leq 45$$
$$x_1, x_2 \geq 0; \quad x_1, x_2 \text{ integers.}$$

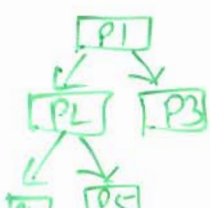
P1 ≡ LP relaxation of above problem (x_1, x_2 integer not required)



Iteration 1: Solve P1 $\Rightarrow x_1 = \frac{15}{4}$ $(= 3.75)$, $x_2 = \frac{9}{4}$, $z = \frac{165}{4}$

Branch on x_1 (any fractional solution) $\Rightarrow (x_1 \geq 4)$ or $(x_1 \leq 3)$.

P2 ≡ P1 + $(x_1 \geq 4)$; P3 ≡ P1 + $(x_1 \leq 3)$



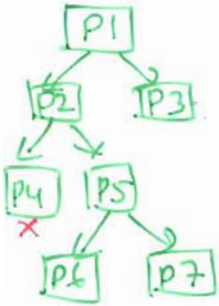
Iteration 2: Solve P2 $\Rightarrow x_1 = 4$, $x_2 = 1.8$, $z = 41$

Branch on $x_2 \Rightarrow (x_2 \geq 2)$ or $(x_2 \leq 1)$

P4 ≡ P2 + $(x_2 \geq 2)$; P5 ≡ P2 + $(x_2 \leq 1)$

Iteration 3: Solve P4 $\Rightarrow$ infeasible; so no further branching

Branch & Bound (Continued)



Iteration 4: Solve P5 $\Rightarrow x_1 = \frac{40}{9}$, $x_2 = 1$, $z = \frac{365}{9}$

Branch on $x_1 \Rightarrow (x_1 \geq 5)$ or $(x_1 \leq 4)$

$P6 \equiv P5 + (x_1 \geq 5)$; $P7 \equiv P5 + (x_1 \leq 4)$

Iteration 5: Solve P6 $\Rightarrow x_1 = 5$, $x_2 = 0$, $z = 40$

A feasible LIP solution found $\Rightarrow$ no further branching

New lower bound = 40

Iteration 6: Solve P7 $\Rightarrow x_1 = 4$, $x_2 = 1$, $z = 37$

No further branching since solution smaller than lower bound

Iteration 7: Solve P3 $\Rightarrow x_1 = x_2 = 3$, $z = 39$

No further branching since solution smaller than lower bound.

Since no branching possible at any node $\Rightarrow$ termination occurs

Optimal solution given by current lower bound : $x_1 = 5$, $x_2 = 0$, $z = 40$

Solving Optimization using Excel Solver

- Enable "Solver" from "Tools" → "Add-in" → "Solver".
(Not needed if "solver" already present under "tools")
- Enter all "data" and "variables"
  - Enter C^T in a row of cells
  - Enter X in a column of cells, with any initial value, say x=
  - Enter formula for $z = C^T X$ in a cell using "mmult(array1, array
and then type "ctrl+shift+enter" to complete
  - Enter A in an array of cells
  - Enter AX in a column of cells using "mmult(array1, array2
  - Enter B in a column of cells
  - Enter lower/upper bounds in columns of cells.
- Start "Solver" from "Tools" → "Solver"
  - Enter z-cell location and select max/min
  - Enter x-cell locations
  - Enter constraints (individually or matrix form)
  - Enter lower/upper bounds
  - Enter integer/binary
  - Click "Solve".
- Optimal solution place in X-column and z-cell.

HW

- Consider the auto mfg. discussed in class, without "OR" constraint
Use Branch & Bound to solve this problem.
(For each LP encountered, use Excel Solver to solve it.)
Draw the Branch-&-Bound tree, labeling each node with LP problem no., and its solution (x-values, and z-value).
- Solve the same problem directly using Solver.