Problem 1. max $100x_1 + 30x_2$

subject to $x_1 + x_2 \le 7$

$10x_1 + 4x_2 \le 40$

$10x_2 \ge 30$

$x_1 \ge 0$

Solution. The constrains of this problem can be changed by introducing slack varibles:

$$x_1 + x_2 + x_3 = 7$$
$$10x_1 + 4x_2 + x_4 = 40$$
$$x_2 - x_5 = 3$$
$$x_i \ge 0 \quad (i=1,2,3,4,5)$$

In order to find the initial feasible solution, we introduce an auxiliary LP Problem as following:

max $(-a_1)$

S.t. $x_1 + x_2 + x_3 = 7$

$10x_1 + 4x_2 + x_4 = 40$

$x_2 - x_5 + a_1 = 3$

$x_i \ge 0,\ a_1 \ge 0 \quad (i=1,2,3,4,5)$

basic variable = variables last place in each eq. $= \{x_3, x_4, a_1\}$.

$\Rightarrow$ Non-basic variable $= \{x_1, x_2, x_5\}$

To find the optimum of auxiliary LP, use the tableau method:

Total varible $=6$ and constraints $=3$.

So the non-basic varibles $=3$ and basic varibles $=3$

We can choose $\{x_3, x_4, a_1\}$ as initial basic variables. Their coefficients in 1st row needs to be zero.

	x_1	x_2	x_3	x_4	x_5	a_1	RHS
(Z)	0	0	0	0	0	-1	0
	1	1	1	0	0	0	7
	10	4	0	1	0	0	40
	0	1	0	0	-1	1	3

First we write the table in terms of basic variables. For this, add last and first row. (Since we want to make the ~~coeff~~ coefficients below for $\{x_3, x_4, a_1\}$ to be zero). This gives the new table from where the optimization begins.

	x_1	x_2	x_3	x_4	x_5	a_1	RHS
(Z)	0	1	0	0	-1	0	3
(4)	1	1	1	0	0	0	7
(5)	10	4	0	1	0	0	40
(6)	0	1	0	0	-1	1	3

The new basic varible are $\{x_2, x_3, x_4\}$ and we get the new tableau by (Z) -(6), (4)-(6), (5) - 4 x (6). :

	x_1	x_2	x_3	x_4	x_5	a_1	RHS
(Z)	0	0	0	0	0	-1	0
(7)	1	0	1	0.	1	-1	4
(8)	10	0	0	1	4	-4	28.
(9)	0	1	0	0	1	-1	3

Since this is optimal (RHS = 0), the current non-basic variables are, $\{x_1, x_5, a_1\}$, so the initial non-basic varibles for the original LP is $\{x_1, x_5\}$, obtained by leaving out the auxiliary variable a_1 from the set $\{x_1, x_5, a_1\}$.

max $100x_1 + 30x_2$.

Subject to: $x_1 + x_2 + x_3 = 7$

$10x_1 + 4x_2 + x_4 = 40$

$x_2 - x_5 = 3$

$x_i \geq 0 \quad (i = 1, 2, 3, 4, 5)$

	x_1	x_2	x_3	x_4	x_5	RHS.
(Z)	100	30	0	0	0	0.
~~(4)~~(10)	1	1	1	0	0	7
(11)	10	4	0	1	0	40
(12)	0	1	0	0	-1	3

We need to make coefficient of basic variables $\{x_2, x_3, x_4\}$ zero in 1st row. So,
Applying (Z) - 30x(12), (10) -(12), (11) - 4 x (12), we made the coefficient of objective $\{x_2, x_3, x_4\}$ zero. we get

	x_1	x_2	x_3	x_4	x_5	RHS
(Z)	100	0	0	0	30	−90
(10)	1	0	1	0	1	4
(11)	10	0	0	1	4	28
(12)	0	1	0	0	−1	3

Adding x_1 into basic varibles and leave x_4 out of the basic varibles

x_1	x_2	x_3	x_4	x_5	RHS
0	0	0	−10	−10	−370
0	0	1	$-\frac{1}{10}$	$\frac{3}{5}$	1.2
1	0	0	$\frac{1}{10}$	$\frac{2}{5}$	2.8
0	1	0	0	−1	3

So the basic varible is $\{x_1, x_2, x_3\}$. and when

$x_1 = 2.8$, $x_2 = 3$, $x_3 = 1.2$ We get the maxmum value 370.