HW #2, EE 653 – SOLUTIONS

Problem 1

A manufacturer of light bulbs is interested in estimating the mean life of the bulbs. Two hundred bulbs are subjected to a reliability test. The bulbs are observed, and the failures in 1,000-hour intervals are recorded as shown in Table 1.1.

Table 1.1. Number of Failures in the Time Intervals

Time Interval (Hours)	*Failures in the Interval*
0–1,000	100
1,001–2,000	40
2,001–3,000	20
3,001–4,000	15
4,001–5,000	10
5,001–6,000	8
6,001–7,000	7
Total	200

Plot the failure density function estimated from data $f_e(t)$, the hazard rate function estimated from data $h_e(t)$, the cumulative probability function estimated from data $F_e(t)$, and the reliability function estimated from data $R_e(t)$. The subscript e refers to *estimated*. Comment on the hazard-rate function.

We estimate $f_e(t)$, $h_e(t)$, $R_e(t)$, and $F_e(t)$ by using the following equations:

$$f_e(t) = \frac{n_f(t)}{n_o\,\Delta t}, \tag{1.15}$$

$$h_e(t) = \frac{n_f(t)}{n_s(t)\,\Delta t}, \tag{1.16}$$

$$R_e(t) = \frac{f_e(t)}{h_e(t)}, \tag{1.17}$$

and

$$F_e(t) = 1 - R_e(t). \tag{1.18}$$

Note that $n_s(t)$ is the number of surviving units at the beginning of the period Δt. Summaries of the calculations are shown in Tables 1.2 and 1.3. The plots are shown in Figures 1.1 and 1.2.

Table 1.2. Calculations of $f_e(t)$ and $h_e(t)$

Time Interval (Hours)	*Failure Density* $f_e(t) \times 10^{-4}$	*Hazard Rate* $h_e(t) \times 10^{-4}$
0–1,000	$\frac{100}{200 \times 10^3} = 5.0$	$\frac{100}{200 \times 10^3} = 5.0$
1,001–2,000	$\frac{40}{200 \times 10^3} = 2.0$	$\frac{40}{100 \times 10^3} = 4.0$
2,001–3,000	$\frac{20}{200 \times 10^3} = 1.0$	$\frac{20}{60 \times 10^3} = 3.33$
3,001–4,000	$\frac{15}{200 \times 10^3} = 0.75$	$\frac{15}{40 \times 10^3} = 3.75$
4,001–5,000	$\frac{10}{200 \times 10^3} = 0.5$	$\frac{10}{25 \times 10^3} = 4.0$
5,001–6,000	$\frac{8}{200 \times 10^3} = 0.4$	$\frac{8}{15 \times 10^3} = 5.3$
6,001–7,000	$\frac{7}{200 \times 10^3} = 0.35$	$\frac{7}{7 \times 10^3} = 10.0$

As shown in Figure 1.1, the hazard rate is constant until time $t = 6{,}000$ hours and then increases with t. Thus $h_e(t)$ can be expressed more or less as

$$h_e(t) = \begin{cases} \lambda_0 & 0 \le t \le 6{,}000 \\ \lambda_1 t & t > 6{,}000 \end{cases},$$

where λ_0 and λ_1 are constants.

Problem 1, Continued

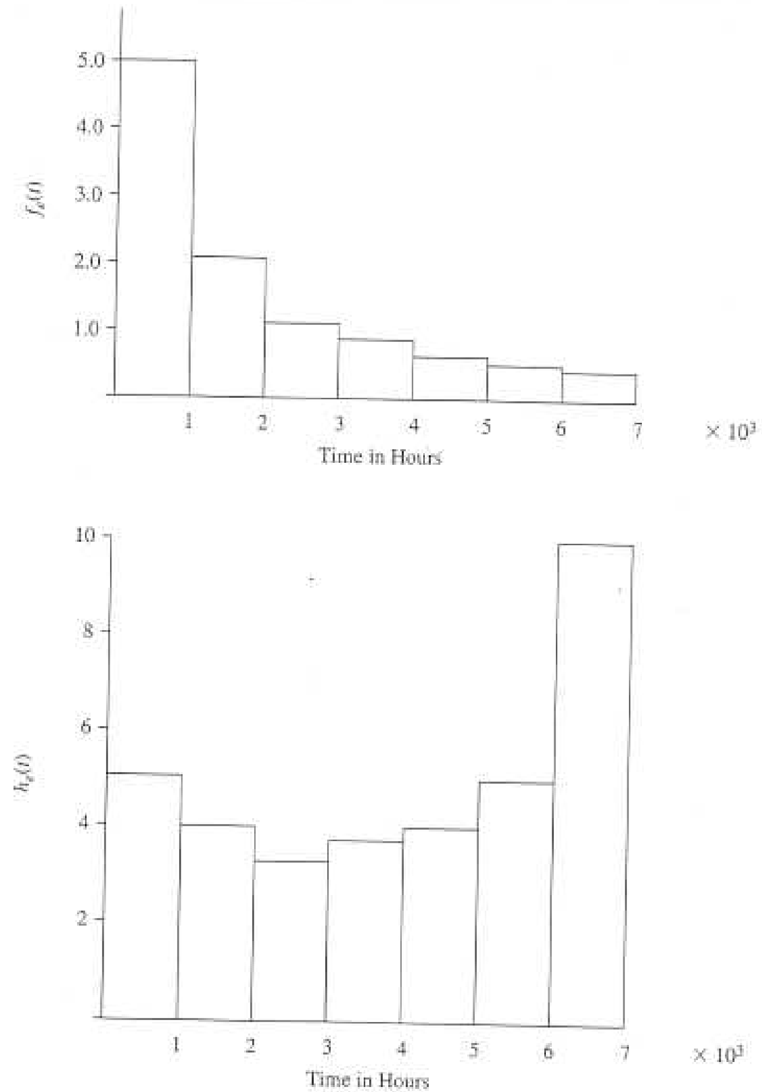

Figure 1.1. Plots of $f_e(t)$ and $h_e(t)$

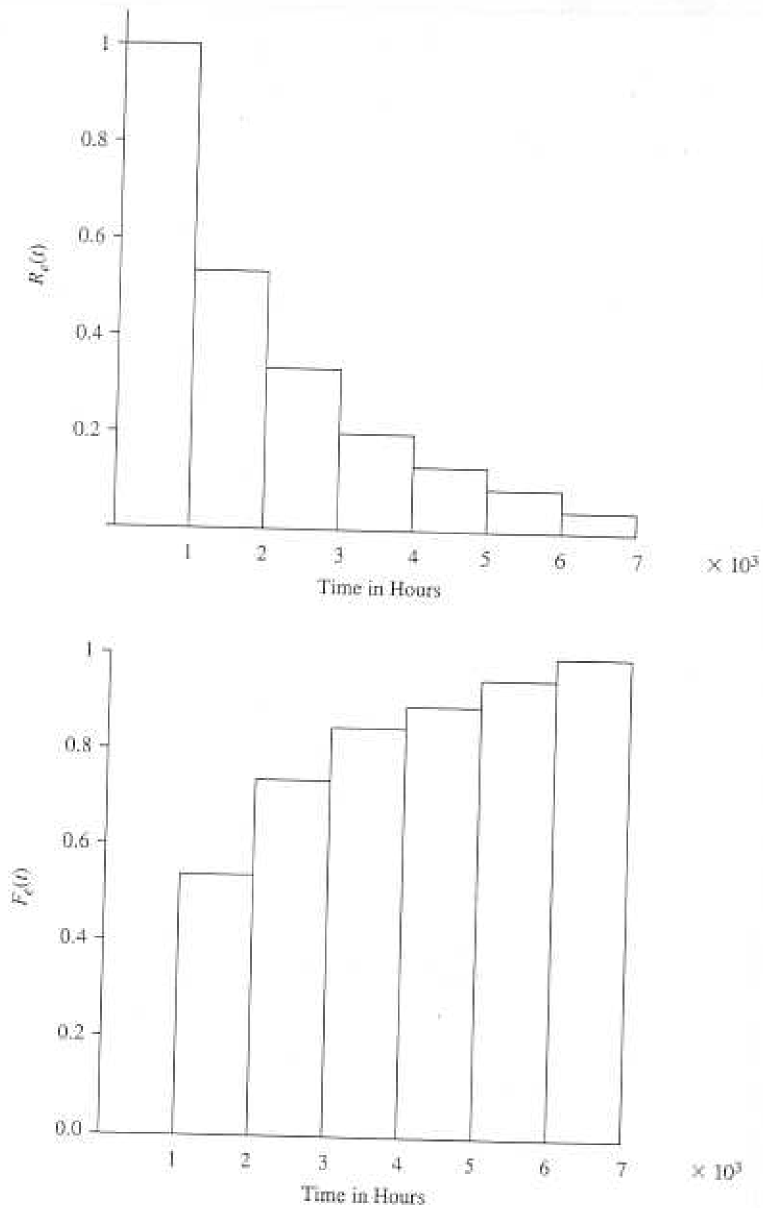

Figure 1.2. Plots of $R_e(t)$ and $F_e(t)$

3

Problem 1 (Continued)

Table 1.3. Calculations of $R_e(t)$ and $F_e(t)$

Time Interval	*Reliability* $R_e(t) = f_e(t)/h_e(t)$	*Unreliability* $F_e(t) = 1 - R_e(t)$
0–1,000	$\frac{5.0}{5.0} = 1.000$	0.000
1,001–2,000	$\frac{2.0}{4.0} = 0.500$	0.500
2,001–3,000	$\frac{1.0}{3.33} = 0.300$	0.700
3,001–4,000	$\frac{0.75}{3.75} = 0.200$	0.800
4,001–5,000	$\frac{0.5}{4.0} = 0.125$	0.875
5,001–6,000	$\frac{0.4}{5.3} = 0.075$	0.925
6,001–7,000	$\frac{0.35}{10.0} = 0.035$	0.965

The above example shows the hazard-rate function is constant for a period of time and then linearly increases with time. In other situations the hazard-rate function may be decreasing, constant, or increasing and the rate at which the function decreases or increases may be constant, linear, polynomial, or exponential with

Problem 2

A manufacturer performs an Operational Life Test (OLT) on ceramic capacitors and finds that they exhibit constant failure rate (used interchangeably with hazard rate) with a value of 3×10^{-8} failures per hour. What is the reliability of a capacitor after one year (10^4 hours)? In order to accept a large shipment of these capacitors, the user decides to run a test for 5,000 hours on a sample of 2,000 capacitors. How many capacitors are expected to fail during the test?

Using Eqs. (1.21) and (1.25), we obtain

$$h(t) = 3 \times 10^{-8} \text{ failures per hour,}$$

and

$$R(t) = e^{-\int_0^t 3\times 10^{-8} dt} = e^{-3\times 10^{-8} t}, \text{ and}$$

$$R(10^4) = e^{-3\times 10^{-4}} = 0.99970.$$

To determine the expected number of failed capacitors during the test, we define the following:

n_o number of capacitors under test,

n_s expected number of surviving capacitors at the end of the test, and

n_f expected number of failed capacitors during the test.

Thus,

$$n_s = e^{-3\times 10^{-8}\times 5000} \times 2000 = 1999 \text{ capacitors and}$$

$$n_f = 2000 - 1999 = 1 \text{ capacitor.}$$

(4)

Problem 4

Permanent magnet synchronous motor (PMSM) brushless DC (BLDC) servos are becoming attractive replacements for DC motors in industrial servo motors. The PMSM BLDC servo has higher torque and velocity bandwidth and does not require the regular brush and maintenance requirements of conventional motors.

A producer of the PMSMs designs a reliability test by subjecting a motor to a continuous load. Upon failure, the motor is immediately repaired and restored to its initial condition. The test is then continued and the above procedure is repeated. The failure and the repair time distributions are exponential with rates λ and μ with estimates of 6×10^{-5} failures per hour and 4×10^{-2} repairs per hour, respectively.

Determine the expected number of motor's failures during $(0, 2 \times 10^4$ hours) and the availability of the motor at the end of two years of testing. Plot $M(t)$ and $A(t)$ for different values of λ and μ.

Since failure and repair times are exponential, following Example 7.3 we have

$$m^*(s) = \frac{\lambda\mu}{(\lambda+\mu)s} - \frac{\lambda\mu}{(\lambda+\mu)^2}\cdot\frac{1}{(s+\lambda+\mu)}$$

and

$$m(t) = \frac{\lambda\mu}{\lambda+\mu} - \frac{\lambda\mu}{(\lambda+\mu)}e^{-(\lambda+\mu)t}.$$

The expected number of renewals in $(0, t]$ is

$$M(t) = \frac{\lambda\mu}{(\lambda+\mu)}t - \frac{\lambda\mu}{(\lambda+\mu)^2} + \frac{\lambda\mu}{(\lambda+\mu)^2}e^{-(\lambda+\mu)t}.$$

Substitution of the values of λ and μ in the above expression results in

$$M(t) = 5.991 \times 10^{-5}t - 0.001495 + 1.495 \times 10^{-3}e^{-4.006\times 10^{-2}t}.$$

The expected number of failures in a 2×10^4 hours interval is

$$M(2 \times 10^4) = 1.197 \text{ failures.}$$

The availability of the motor is

$$A(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t},$$

and the availability at the end of two years of testing is

$$A(2 \times 10^4) = \frac{4 \times 10^{-2}}{4.006 \times 10^{-2}} + \frac{6 \times 10^{-5}}{4.006 \times 10^{-2}}e^{-(4.006)\times 2\times 10^2}$$

or

$$A(2 \times 10^4) = 0.9985.$$

The plots of $M(t)$ and $A(t)$ for different values of λ and μ are shown in Figures 7.3 and 7.4, respectively.

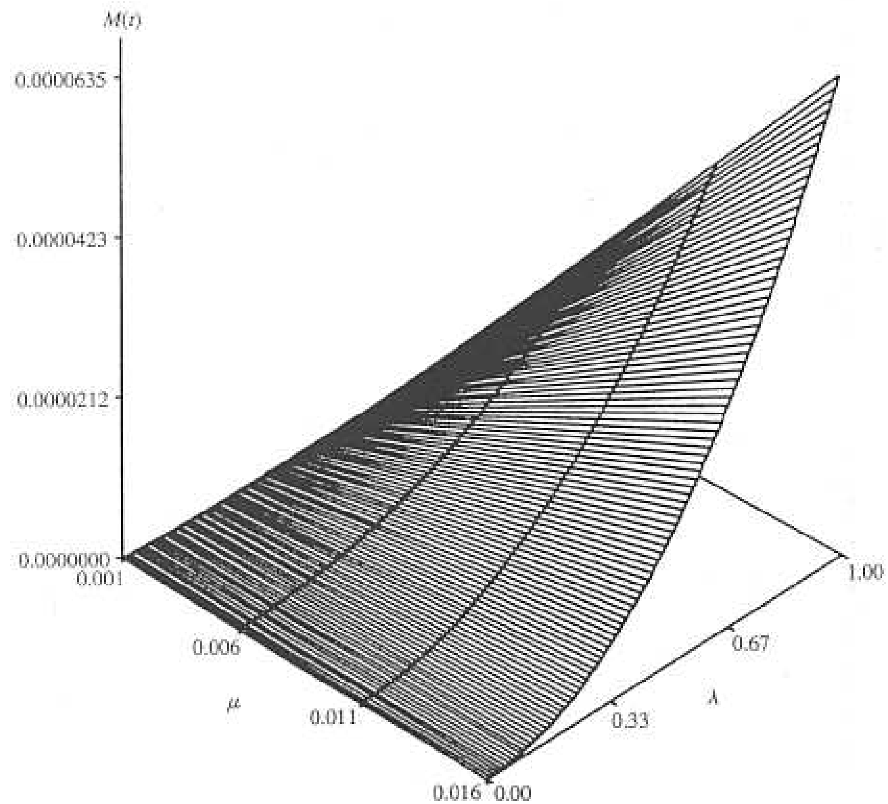

Figure 7.3. $M(t)$ for Different λ and μ

Problem 4 (Continued)

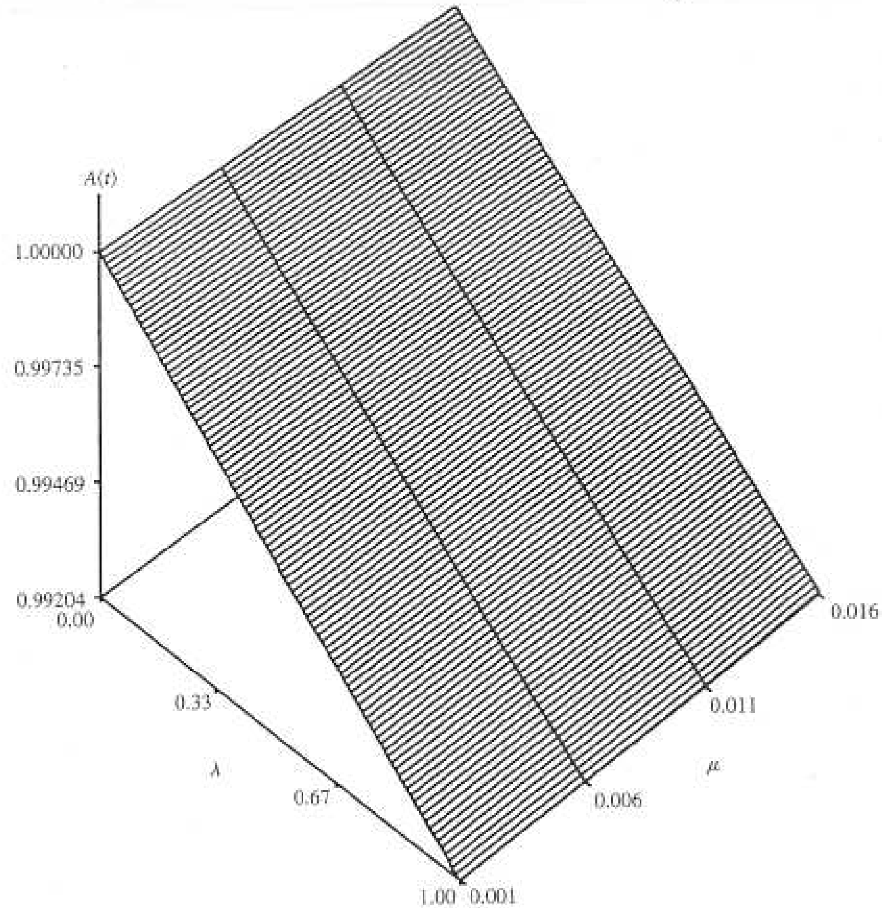

Figure 7.4. $A(t)$ for Different λ and μ