Stochastic Programming and its Application to Optimal Generation Planning

I. Statement of the Learning Objectives

a) Introduce formulation of stochastic programming problems and solution algorithms.

b) Illustrate the application of stochastic programming to optimal generation planning.

II. Introduction of Stochastic Programming

Stochastic programming deals with the theory and methods of incorporating stochastic variations into a mathematical programming problem. The area of stochastic programming was created in the middle of the last century, following fundamental achievements in mathematical programming. Generally speaking, a mathematical programming problem is known as follows

$$\min\ g_0(x) \qquad (1)$$

s.t.

$$g_i(x) \le 0,\quad i = 1, \cdots, m$$

$$x \in X \subset R^n$$

where x is the vector of decision variables, g_0 is the objective function and the functions g_i ($i = 1, \cdots, m$) are the constraints.

Depending on the properties of the functions g_i and the set X, problem (1) is called

(1) linear, if the set X is a convex polyhedral and the functions g_i ($i = 0, \cdots, m$) are linear;

(2) nonlinear, if at least one of the functions g_i ($i = 0, \cdots, m$) is nonlinear or X is a non-convex polyhedral set.

If some of the $x_j \in X$ are required to be integers, then (1) becomes a (mixed) integer programming problem.

In many situations, it is unreasonable to assume that the functions g_i and the the X in (1) are deterministically fixed. For example, the future load demand in a power system is often modeled as an uncertain parameter associated with a probability distribution. Therefore, problem (1) may not be the appropriate model for describing the real life decision problem we want to solve. Random parameters in (1) may lead to the following stochastic programming problem

$$\text{"min"}\; g_0(x, \tilde{\xi}) \tag{2}$$

subject to

$$g_i(x, \tilde{\xi}) \le 0, \quad i = 1, \cdots, m$$
$$x \in X \subset R^n$$

where $\tilde{\xi}$ is a random vector varying over a set $\Xi \subset R^k$. More precisely, we assume that F is a family of "events", i.e. subsets of Ξ. The probability distribution P on F is given. Therefore, for every subset $A \subset \Xi$ that is an event, i.e. $A \in F$, the probability $P(A)$ is known. Furthermore, we assume that the functions $g_i(x, \bullet) : \Xi \mapsto R \; \forall x, i$ are random variables themselves, and that the probability distribution P is independent of x. The observation that some parameters in real life optimization problems could be random dates back to the 1950s.

III. An Illustrative Example

Reference (13) provides an illustrative example showing the formulation and solution of a stochastic linear programming problem.

A. A Deterministic Linear Programming Problem

A refinery can simultaneously produce two different goods, *prod1* and *prod2* using two raw materials *raw1* and *raw2*. The output of products per unit of the raw materials and the unit costs of the raw materials $c = (c_{raw1}, c_{raw2})^T$, the demands for the products $h = (h_{prod1}, h_{prod2})^T$ and the production capacity or the maximal total amount of raw materials that can be processed b, are given in Table I.

Table I. Productivities of a refinery π(*rawi*, *prodj*).

	Products			
Raws	*prod1*	*prod2*	*c*	*b*
raw1	2	3	2	1
raw2	6	3	3	1
relation	≥	≥	=	≤
h	180	162	γ	100

Based on the given information, a linear programming problem can be formulated as follows

$$\min(2x_{raw1} + 3x_{raw2}) \tag{3}$$

s.t.

$$x_{raw1} + x_{raw2} \le 100$$
$$2x_{raw1} + 6x_{raw2} \ge 180$$
$$3x_{raw1} + 3x_{raw2} \ge 162$$
$$x_{raw1} \ge 0$$
$$x_{raw2} \ge 0$$

We can give a graphical representation of the set of feasible production plan as shown in Fig. 1. Given the cost function $\gamma(x) = 2x_{raw1} + 3x_{raw2}$, we can easily found the unique optimal solution to (3) as

$$x^*_{raw1} = 36 \tag{4}$$
$$x^*_{raw2} = 18$$
$$\gamma(x^*) = 126$$

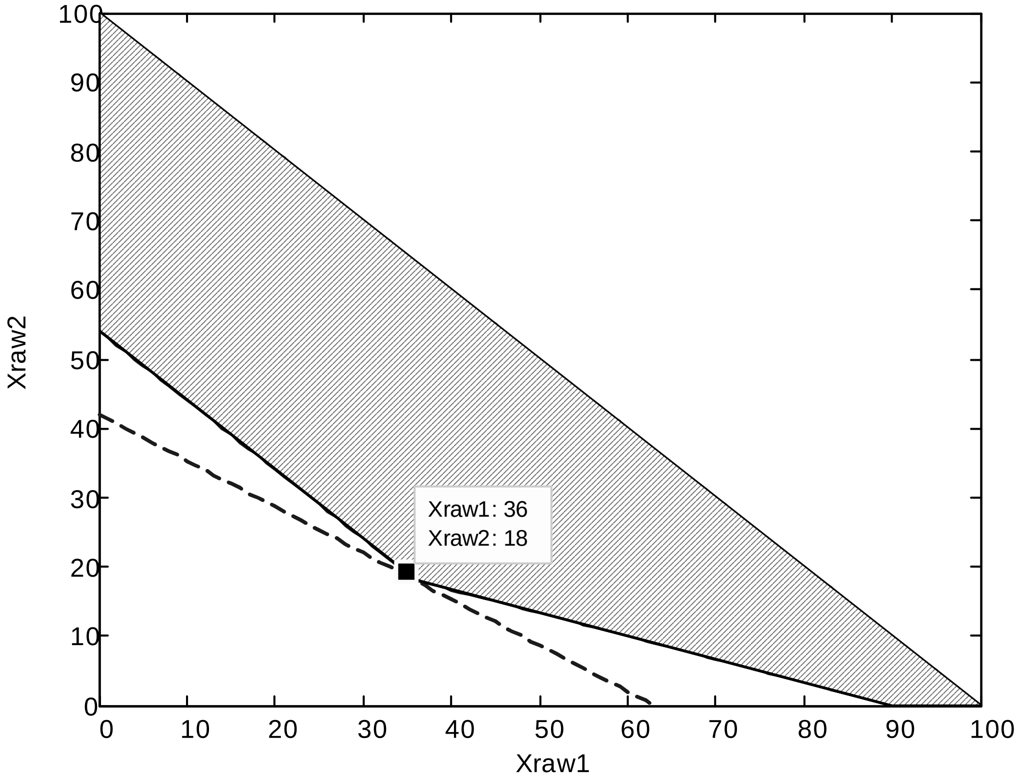

Fig. 1. Set of feasible production plans of the deterministic LP.

B. A Stochastic Linear Programming Problem with Fixed Recourse

The above production problems is properly described by (3) and solved by (4) provided the productivities, the unit costs, the demands and the capacity in Table I are fixed data and known to us prior to making the decision on the production plan. However, this is not always a realistic assumption. It may happen that at least some of the data, productivities and demands for example, can vary randomly and **we have to make the decision on the production plan before knowing the exact values of those data**.

To be more specific, let us assume that

- the model describes the weekly production process of a refinery relying on two countries for the supply of crude oil (*raw1* and *raw2* , respectively), supplying one big company with gasoline (*prod1*) for its distribution system of gas stations and another with fuel oil (*prod2*) for power plants;
- it is known that the productivities π(*raw1*, *prod1*) and π(*raw2, prod2*), i.e. the output of gas from *raw1* and the output of fuel form *raw2* may change randomly (whereas the other productivities are deterministic);
- the weekly demands of the clients, h_{prod1} for gas and h_{prod2} for fuel are

varying randomly;

- the weekly production plan (x_{raw1}, x_{raw2}) has to be fixed in advance and cannot be changed during the week;
- the clients expect their actual demand to be satisfied during the corresponding week.

Assume that, owing to statistics, we know that

$$h_{prod1} = 180 + \tilde{\varsigma}_1, \qquad (5)$$

$$h_{prod2} = 162 + \tilde{\varsigma}_2,$$

$$\pi(raw1, prod1) = 2 + \tilde{\eta}_1,$$

$$\pi(raw2, prod2) = 3.4 - \tilde{\eta}_2.$$

where the random variables $\tilde{\varsigma}_j$ are modeled using normal distributions, and $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are distributed uniformly and exponentially respectively, with the following parameters:

$$\tilde{\varsigma}_1 \sim N(0,12), \qquad (6)$$

$$\tilde{\varsigma}_2 \sim N(0,9),$$

$$\tilde{\eta}_1 \sim UINF(-0.8,0.8),$$

$$\tilde{\eta}_2 \sim EXP(2.5).$$

For simplicity, we assume that these four random variables are mutually independent. Since the random variables $\tilde{\varsigma}_1$, $\tilde{\varsigma}_2$ and $\tilde{\eta}_2$ are unbounded, we restrict our considerations to their respective 99% confidence intervals. So the realizations of the above random variables are

$$\varsigma_1 \in [-30.91, 30.91], \qquad (7)$$

$$\varsigma_2 \in [-23.18, 23.18],$$

$$\eta_1 \in [-0.8, 0.8],$$

$$\eta_2 \in [0.0, 1.84].$$

Geometrically, the consequence of the random parameter changes may be very complex. The effect of only the right-hand sides ς_i varying over the intervals given in (7) corresponds to parallel translations of the corresponding facets of the feasible set as shown in Fig. 2.

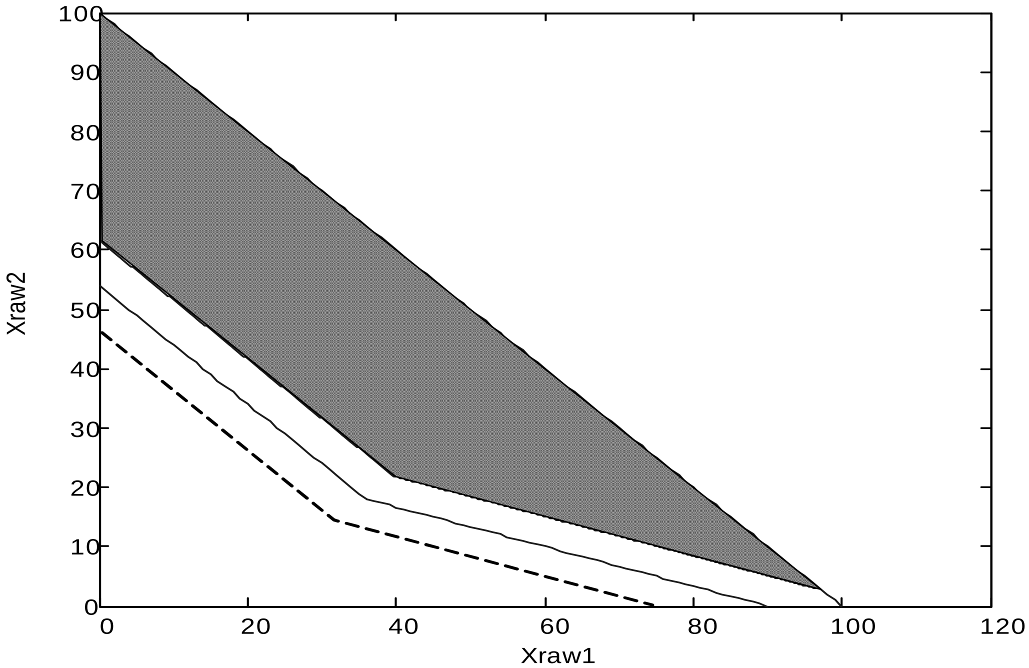

Fig. 2. Feasible set varying with demands.

We may instead consider the effect of only the η_i changing their values within the intervals. That results in rotations of the related facets as shown in Fig. 3, where the centers of rotation are indicated by small circles.

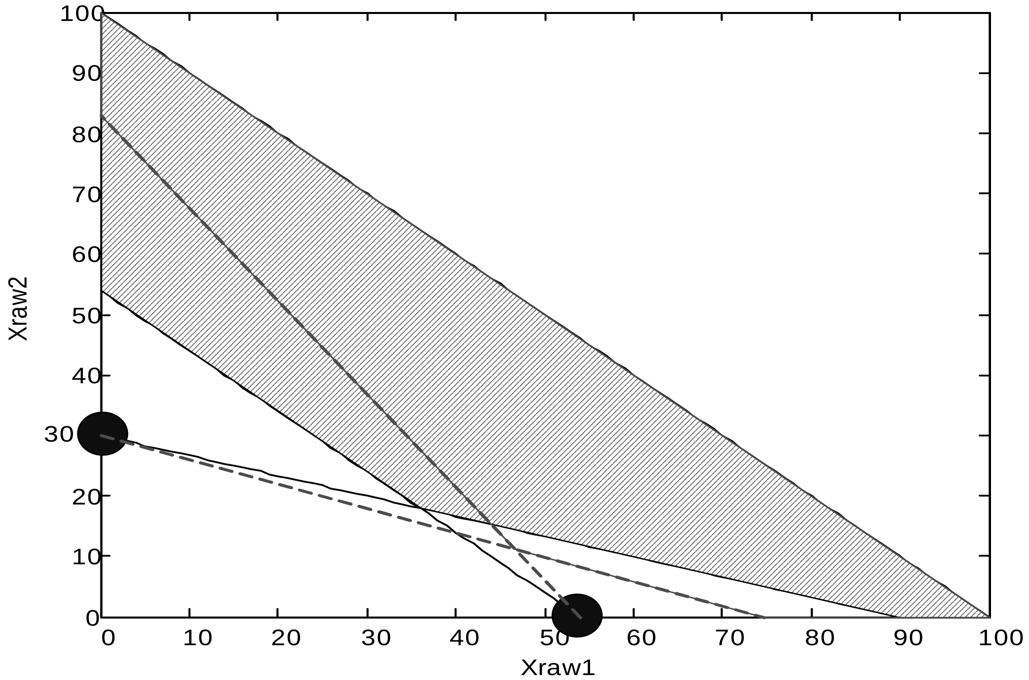

Fig. 3. Feasible set varying with productivities.

Assume that if the demands cannot be covered by the production, there will be a “penalty” costs to the refinery. The amount of shortage has to be bought from the market. These penalties are supposed to be proportional to the respective shortage in products. Assume the penalties of per unit of undeliverable products are

$q_{prod1} = 7$, $q_{prod2} = 12$

The costs due to shortage of production or in general due the amount of violation in the constraints are actually determined after the observation of the random data and are denoted as ***recourse costs***. In a case of repeated execution of the production plan it makes sense to apply and ***expected value criterion***.

More precisely, we may want to find a production plan that minimizes the sum of the original *first-stage* (i.e. production) costs and the *expected recourse costs*. To formalize this approach, we introduce for each of the two stochastic constraints a *recourse variable* $y_i(\tilde{\xi})$, *i*=1, 2, which simply measures the corresponding shortage in production if there is any. The deterministic programming problem (3) can be replaced by the ***stochastic programming with recourse***

$$\min\{2x_{raw1} + 3x_{raw2} + E_{\tilde{\xi}}[7y_1(\tilde{\xi}) + 12y_2(\tilde{\xi})]\} \qquad (10)$$

s.t.

$$x_{raw1} + x_{raw2} \le 100,$$

$$\alpha(\tilde{\xi})x_{raw1} + 6x_{raw2} + y_1(\tilde{\xi}) \ge h_1(\tilde{\xi}),$$

$$3x_{raw1} + \beta(\tilde{\xi})x_{raw2} + y_2(\tilde{\xi}) \ge h_2(\tilde{\xi}),$$

$$x_{raw1} \ge 0,$$

$$x_{raw2} \ge 0,$$

$$y_1(\tilde{\xi}) \ge 0,$$

$$y_2(\tilde{\xi}) \ge 0.$$

where

$h_1(\tilde{\xi}) = h_{prod1} = 180 + \tilde{\varsigma}_1$,

$h_2(\tilde{\xi}) = h_{prod2} = 162 + \tilde{\varsigma}_2$,

$\alpha(\tilde{\xi}) = \pi(raw1, prod1) = 2 + \tilde{\eta}_1$,

$\beta(\tilde{\xi}) = \pi(raw2, prod2) = 3.4 - \tilde{\eta}_2$.

In (10) $E_{\tilde{\xi}}$ represents the expected value with respect to the distribution of $\tilde{\xi}$. In general, the stochastic constraints have to hold almost surely (*a.s.)* (i.e., they are to be satisfied with probability 1).

If $\tilde{\xi}$ has a finite discrete distribution $\{(\xi^i, p_i), i = 1, \cdots, r\}$ $(p_i > 0\ \forall i)$ then (10) is just an ordinary linear programming with a so called **dual decomposition structure**

$$\min\{2x_{raw1} + 3x_{raw2} + \sum_{i=1}^{r} p_i[7y_1(\xi^i) + 12y_2(\xi^i)]\} \qquad (11)$$

s.t.

$$x_{raw1} + x_{raw2} \le 100,$$
$$\alpha(\xi^i)x_{raw1} + 6x_{raw2} + y_1(\xi^i) \ge h_1(\xi^i)\ \ \forall i,$$
$$3x_{raw1} + \beta(\xi^i)x_{raw2} + y_2(\xi^i) \ge h_2(\xi^i)\ \ \forall i,$$
$$x_{raw1} \ge 0,$$
$$x_{raw2} \ge 0,$$
$$y_1(\xi^i) \ge 0\ \ \forall i,$$
$$y_2(\xi^i) \ge 0\ \ \forall i.$$

Depending on the number of realization of $\tilde{\xi}$, *r*, this linear programming may become very large in scale, but its particular block structure is amenable to specially designed algorithms.

Assume that only the demands, $h_i(\tilde{\xi})$, *i*=1,2, are changing their values randomly, whereas the productivities are fixed. This is the cased shown in Fig. 2.

One way to solve this stochastic programming problem is to approximate the normal distributions by discrete ones. Fig. 4 shows the discrete distributions for N(0, 12) and N(0, 9) with 15 realizations each. Therefore, there are total 15^2=225 realizations for the joint distribution, and hence 225 blocks in the decomposition problem of (11). The optimal solution for the linear programming (11) is

$$\tilde{x} = (\tilde{x}_{raw1}, \tilde{x}_{raw2}) = (38.539, 20.539), \quad \gamma(\tilde{x}) = 140.747$$

With corresponding *first-stage costs* of

$$\gamma_I(\tilde{x}) = 2\,\tilde{x}_{raw1} + 3\,\tilde{x}_{raw2} = 138.694$$

Define $\rho(x)$ as the empirical reliability (i.e. the probability to be feasible) for any production plan *x*. For the above solution $\tilde{x}$, the corresponding reliability is

$$\rho(\tilde{x}) = 0.9115$$

However, using the solution of the original linear programming $x^*(36,18)$ would yield the total expected cost $\gamma(x^*) = 199.390$ and an empirical reliability of $\rho(x^*) = 0.3188$.

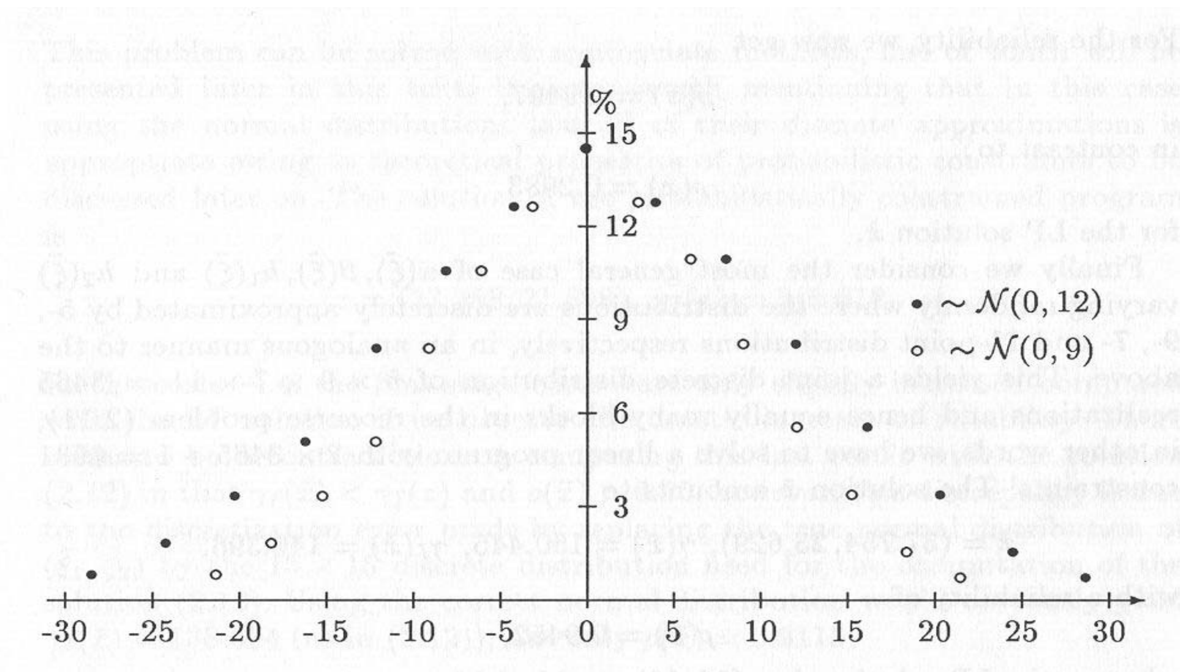

Fig. 4 Discrete distribution generated from *N*(0,12), *N*(0,9); (r_1,r_2)=(15,15).

C. A Stochastic Linear Programming Problem with Probabilistic Constraints

Assume the management of the refinery is convinced that it is absolutely necessary to maintain a reliability of 95% with respect to satisfying the clients' demands. In this case we may formulate the following stochastic programming with ***joint probabilistic constraints***:

$$\min(2x_{raw1} + 3x_{raw2}) \tag{12}$$

s.t.

$$x_{raw1} + x_{raw2} \le 100$$
$$x_{raw1} \ge 0$$
$$x_{raw2} \ge 0$$
$$P\begin{pmatrix} 2x_{raw1} + 6x_{raw2} \ge h_1(\tilde{\xi}) \\ 3x_{raw1} + 3x_{raw2} \ge h_2(\tilde{\xi}) \end{pmatrix} \ge 0.95$$

The solution of the probabilistically constrained programming is
$z = (37.758, 21.698)$, $\gamma_I(z) = 140.612$

IV. General Formulation and Solution of Stochastic Programming

A. General Formulation and Solution of the Two-Stage Stochastic Linear Programming with Fixed Recourse

The general formulation of the two-stage stochastic linear programming with fixed recourse is

$$\min_x E_{\tilde{\xi}}\{c^T x + Q(x, \tilde{\xi})\} \tag{13}$$

s.t.

$$Ax = b$$
$$x \ge 0$$

where

- $Q(x,\xi) = \min\{q^T y \mid Wy = h(\xi) - T(\xi)x,\ y \ge 0\}$ is the *recourse function*,
- y is the recourse vector,
- W is the *fixed recourse matrix*,
- c is the unit cost,

- q is the penalty.

If the random data have a joint continuous distribution, discrete approximation can be used to solve the above problem which was first suggested by Dantzig and Madansky [2].

If the random data have a joint finite discrete probability distribution $\{(\xi^k, p_k), k=1,\cdots,r\}$. Then problem (13) becomes a ***linear programming* having *dual-decomposition structure***

$$\min_x \{c^T x + \sum_{k=1}^{r} p_k q^T y^k\} \tag{14}$$

s.t.

$$Ax = b$$
$$x \geq 0$$
$$T(\xi^k)x + Wy^k = h(\xi^k), k = 1,\cdots,r$$
$$y^k \geq 0$$

where p_k is the joint probability for the realization of the random vector ξ^k. A basic solution method for this linear programming problem is the **dual decomposition method** [13] which is a well-known method to solve a sort of linear programming problems.

B. General Formulation and Solution of the Stochastic Linear Programming with Probabilistic (Chance) Constraints

The general formulation of the stochastic linear programming with *joint probabilistic constraints* is as follows

$$\min_{x\in X} E_{\tilde{\xi}} c^T(\tilde{\xi})x \tag{15}$$

s.t.

$$P(\{T(\xi)x \geq h(\xi)\}) \geq \alpha$$

where

$\alpha \in [0,1]$

Because of the probabilistic aspect of the stochastic linear programming with chance constraints, it is possible to find deterministic constraints which are equivalent to the chance constraints.

V. The Application of Stochastic Programming to Optimal Generation Planning

In this section, we study the problem of optimal generation planning using stochastic programming. The reasons which justify the use of a multistage stochastic model are discussed. Then a formulation for such a model is presented. At last, a two-stage model as a study case is derived.

A. The Problem of Generation Planning

The aim of generation planning is to find the most economical generation expansion scheme achieving a certain reliability level according to the load forecast during a certain period of time [11]. The following questions need to be answered:

1. When to invest in new generating units?
2. Where to invest in new generating units?
3. What type of generating units to install?
4. What capacity of generating units to install?

There are three kinds of generation planning methods:

1. Static assessment method
2. Dynamic assessment method
3. Stochastic assessment method

B. Static Assessment Method

The level of demand varies over time. The electricity producers usually represent the demand in terms of a so-called “load duration curve” which describes the total time through a certain period that the load exceeds a value d as shown in Fig. 5. A procedure is given in [12] to obtain the load duration curve from the instantaneous load curve.

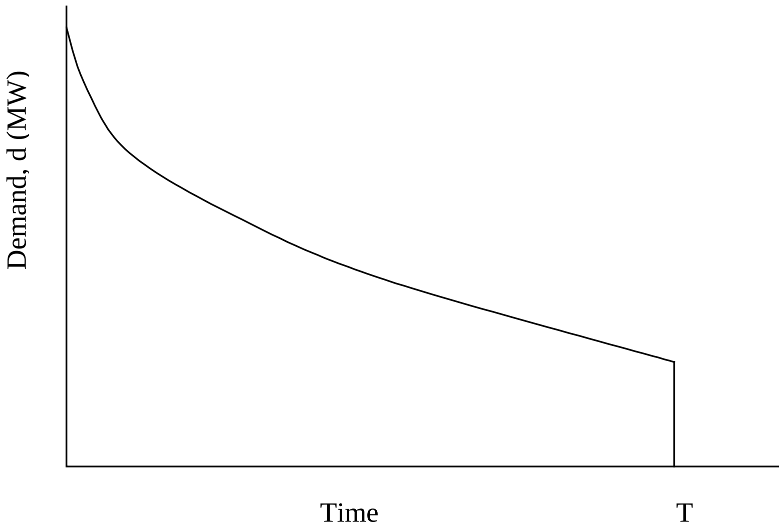

Fig. 5. Load duration curve.

The load duration curve can be approximated by a piecewise constant curve with *k* segments as shown in Fig. 6. Let *d1=D1*, *dj=Dj-Dj-1*, *j=2,…,k* represent the additional power required in the so-called "mode *j*" for a duration T_j. In the static assessment model, the objective is to find the optimal investment for each mode *j*, i.e. that one which minimizes the total cost of producing 1 MW of electricity during time T_j.

$$i(j) = \arg\min_{i=1,\ldots,n}\{\frac{c_i + q_i T_j}{a_i}\} \quad (17)$$

where

- *n* is the number of available generation categories
- c_i is the investment cost of a given power plant *i*
- q_i is the operating cost
- a_i is the availability factor which indicates the percent of time the power plant can effectively be operated

From the above formulation of the static generation planning problem, we find that the base load demand (associated with large values of T_j, i.e. small indices *j*) should be covered by equipments with low operating costs (scaled by availability factor a_i) while peak-load demand (associated with small values of T_j, i.e. large indices *j*) should be covered by equipments with low investment costs (also scaled by their availability factor).

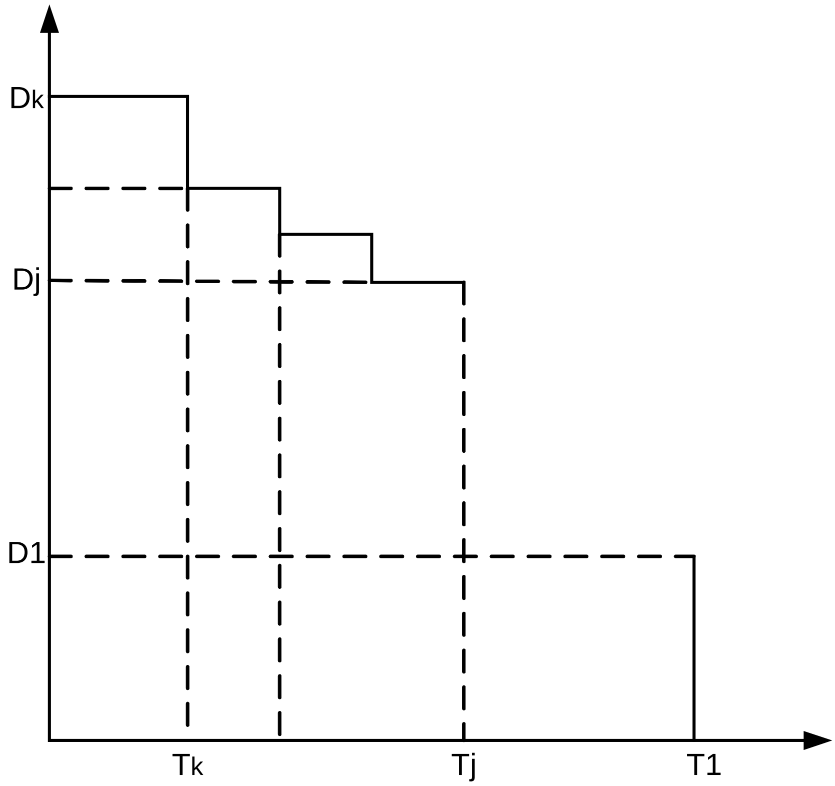

Fig. 6. Approximated load duration curve.

C. Dynamic Assessment Method

The static assessment method is simple and direct. However, it can not consider the following factors:

- the long-term evolution of equipment costs,
- the long-term evolution of the load curve,
- the appearance of new technologies,
- the obsolescence of presently available equipment.

All of the above factors indicate that it is not optimal in the long run to invest only in view of the short-term ordering of equipment given by (17). Therefore, a long-term optimal policy should be considered. The dynamic *N*-stage generation planning problem can be formulated as

$$\min_{x,y,s} \sum_{t=1}^{N} \Big(\sum_{i=1}^{n} c_i^t s_i^t + \sum_{i=1}^{n} \sum_{j=1}^{k} q_i^t T_j^t y_{ij}^t \Big) \tag{18}$$

s.t.

$$s_i^t = s_i^{t-1} + x_i^t - x_i^{t-L_i} \quad i = 1, \cdots, n, \quad t = 1, \cdots, N \tag{19}$$

$$\sum_{i=1}^{n} y_{ij}^{t} = d_{j}^{t} \qquad j = 1, \cdots, k, \quad t = 1, \cdots, N \tag{20}$$

$$\sum_{j=1}^{k} y_{ij}^{t} \le a_{i}(g_{i}^{t} + s_{i}^{t}) \qquad i = 1, \cdots, n, \quad t = 1, \cdots, N \tag{21}$$

$$s, y, x \ge 0$$

where

- n is number of generation categories available,
- x_i^t is new capacity made available for generation of category i at time t,
- s_i^t is total capacity of generation of category i available at time t,
- a_i is availability factor of generation of category i,
- L_i is life-time of generation of category i,
- g_i^t is existing capacity of generation of category i at time t, decided before t=1,
- d_j^t is maximal power demanded in mode j at time t,
- T_j^t is duration of mode j at time t,
- y_{ij}^t is capacity of generation of category i effectively used at time t in mode j,
- c_i^t is unit investment cost for generation of category i at time t (on a yearly equivalent basis),
- q_i^t is unit production cost for generation of category i at time t.

The objective function (18) is the sum of the investment plus maintenance costs and operating cost. The decision variables in each period t include new capacities x_i^t made available in each generation category and capacities y_{ij}^t operated in each mode for each generation category.

Newly decided capacities x_i^t increase the total capacity s_i^t decided after t=1, as given by (19) where the obsolescence of equipments after their lifetime is also considered. It is assumed that $x_i^\tau = 0$ if $\tau < 0$.

By (20), the optimal operation of equipments must be chosen in such a way as to meet load demand in all modes.

Inequality constraint (21) indicates that available capacities depend on capacities g_i^t decided before t=1, the capacities s_i^t decided after t=1 and the availability factor.

D. Stochastic Assessment Method

The evolution of equipment costs, in particular fuel costs, the evolution of total demand, the date of appearance of new generation technologies and the lifetime of existing equipments can all be considered as random variables. In such situation, a stochastic generation planning model is needed. For simplicity, only uncertainty of demand and costs are considered in the following discussion.

The stochastic model is

$$\min E_\xi \sum_{t=1}^{N} (\sum_{i=1}^{n} c_i^t s_i^t + \sum_{i=1}^{n} \sum_{j=1}^{k} q_i^t T_j^t y_{ij}^t) \qquad (22)$$

$$s_i^t = s_i^{t-1} + x_i^t - x_i^{t-L_i} \qquad (23)$$

$$\sum_{i=1}^{n} y_{ij}^t = d_j^t \qquad (24)$$

$$\sum_{j=1}^{k} y_{ij}^t \le a_i (g_i^t + s_i^{t-\Delta_i}) \qquad (25)$$

$$a_n (g_n^t + s_n^{t-1} + x_n^t) \ge D_m^t - \sum_{i=1}^{n-1} a_i (g_i^t + s_i^{t-\Delta_i}) \qquad (26)$$

$$s, x, y \ge 0$$

where

- x_i^t is new capacity decided at time *t* for equipment *i*, $i = 1, \cdots, n$,
- s_i^t is total capacity of *i* available plus in order at time *t*,
- n is a generation category associated with $\Delta_n = 0$,

- ξ_t represents random variables at time *t*,

and other variables are as before. The elements of ξ_t are the load demands $(d_1^t,\cdots,d_n^t)$ and the costs (c^t,q^t). The decision vectors (x^t,s^t,y^t) are conditional on the realizations $(\xi_1,\cdots,\xi_t)$.

In the stochastic generation planning model, it is assumed that there exists a generation category with high operating costs and zero construction delay for the purpose of the relatively complete recourse property of the model. For any period t and any realization ξ of the random event, an investment is made in that generation category, which for simplicity is always supposed to be technology n, if the level of capacity investments in the previous periods is insufficient to cover present demand.

When N is small enough, (23) can be simplified as

$$s_i^t = s_i^{t-1} + x_i^t$$

E. Study Case

In order to demonstrate the application of the stochastic generation planning techniques, a tow-stage linear version of (22)-(26) is presented [1]. The example has 3 operating modes, 4 generation categories, one period construction delay for all generation categories, and no equipment available, so $g=(0,0,0,0)$. It is assumed that $d_3=2$, $d_2=3$ and $d_1=\xi$, where ξ can take the value 3, 5 or 7 with probability 0.3, 0.4, and 0.3 respectively. Moreover $T_2=0.6T_1$, $T_3=0.1T_1$ and $T_1=10$. Since *N*=2 and all equipments have a one period construction lead time, (23) reduces to $s_i^t = x_i^t$, so the variables s^t are suppressed from the formulation and the index *t* can be omitted. The constraint (26) takes the simple form $\sum_{i=1}^{4} x_i \ge 12$ where $12=\max_{\xi}(\xi+d_2+d_3)$.

The investment costs for the four generation categories are (10, 7, 16, 6) respectively. Assuming $T_1=10$, the operating costs in mode1 are (40, 45, 32, 55). Then, if T2=6 and T3=1, the following model can be obtained

$$
\begin{aligned}
z = \min 10x_1 + 7x_2 + 16x_3 + 6x_4 + E_\xi \min (40y_{11} + 45y_{21} + 32y_{31} + 55y_{41} \\
+ 24y_{12} + 27y_{22} + 19.2y_{32} + 33y_{42} \\
+ 4y_{13} + 4.5y_{23} + 3.2y_{33} + 5.5y_{43})
\end{aligned} \quad (27)
$$

s.t.

$$
\begin{aligned}
&10x_1 + 7x_2 + 16x_3 + 6x_4 \le 120 \\
&y_{11} + y_{21} + y_{31} + y_{41} \ge \xi \\
&y_{12} + y_{22} + y_{32} + y_{42} \ge 3 \\
&y_{13} + y_{23} + y_{33} + y_{43} \ge 2 \\
&y_{11} + y_{12} + y_{13} \le x_1 \\
&y_{21} + y_{22} + y_{23} \le x_2 \\
&y_{31} + y_{32} + y_{33} \le x_3 \\
&y_{41} + y_{42} + y_{43} \le x_4 \\
&x_1 + x_2 + x_3 + x_4 \ge 12 \\
&x \ge 0 \\
&y \ge 0
\end{aligned}
$$

where ξ can take the value 3, 5, or 7 with probability 0.3, 0.4 and 0.3 respectively.

The optimal solution is given by $x_1 = 8/3$; $x_2 = 4$; $x_3 = 10/3$; $x_4 = 2$ with objective value $z = 381.853$.

V. References

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