Identifying High Risk N-k Contingencies for On-Line Security Assessment

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***Abstract*—It is impractical to analyze all of the possible contingencies in a large-scale interconnected power network. Therefore a standard approach is to analyze only a subset of the contingencies. The normal method of selecting this subset is via use of the so-called N−1 rule. This paper goes a step further by proposing a new method of forming a contingency list, based on substation configuration obtained from topology processing data and probability analysis of protection system failures. This method is particularly suited for on-line security assessment. Protection system failures assessed include stuck breakers and failure to operate. The method is described via application to a single substation, verified using the IEEE-RTS96, and illustrated using topology data from a large utilities EMS.**

***Key Words*—power system, transmission, security assessment, contingency identification, topology data, functional group, probability order, rare event approximation, event tree, high order contingency.**

I. Definitions

A list of terminologies used in the paper follows:

- *Event*: Any occurrence that has a significant impact to power system state. It may be a component outage, operational decision, load change, etc.
- *Contingency*: A specified set of events occurring within a short duration where the first is unexpected, e.g., a fault followed by breaker action and subsequent line removal.
- *N-1 contingency*: A contingency resulting in loss of one component.
- *N-k contingency*: A contingency resulting in loss of k components where it is implicit that $k>1$.
- *Initiating contingency*: A contingency that initiates a cascading sequence.
- *Protection failure:* The failure of a protection system, including relay system and circuit breakers, to perform the action as designed.
- *Functional group:* A group of components that operate and fail together due to their connection structure and protection scheme.
- *Per-demand failure rate*: The conditional probability a component fails to perform the function when it is demanded. This paper focuses on the failure of protection following component faults.

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II. Introduction

There is increasing need to provide operators with enhanced on-line information regarding system security levels, what influences these levels, and what actions should be taken, or not taken, in order to most economically achieve an improved level. This paper aims to address one aspect of this issue by providing a method to identify multiple component contingencies that represent high risk.

The causes of cascading events in power systems are various [1]. One major contribution to cascading is high order initiating contingencies—removal of several power system components in a very short time, typically within seconds. Contingency set identification is an essential step in monitoring the power system security level [1]. Most literature [2][3] on contingency selection emphasizes screening methods to select contingencies from a presumed $N-1$ contingency set plus a limited number of high order contingencies, ranking them using an appropriate severity index. Some exceptions include [4]-[6] which studied the effect of multiple component contingencies caused by substation and protection failures. However, the literature on systematic selection of high order contingencies, called $N-k$ contingencies (where $k \geqslant 2$ is implicit), is limited. [7] and [8] proposed the on-line detection of hidden failure in protection device to prevent cascading failure. The proposed method needs exhaustive information on the logic of protection device installed in power system, which make it very difficult to be implemented. The difficulty of $N-k$ contingency selection lies in its combinatorial nature: the total number of distinct non-ordered (simultaneous) $N-k$ contingencies is $N!/[k!(N-k)]$. For a very modest size power system model with N=1000, there are 499,500 $N-2$ contingencies, 166,167,000 $N-3$ contingencies, over 41 billion $N-4$ contingencies, and so on. One might argue that most of these contingencies are so low in probability that they do not warrant attention. However, $N-k$ contingencies do occur, and when they do, consequences can be very severe, and these very practical facts motivate the objective of this paper, to identify high risk N-k contingencies for on-line security assessment. Such contingencies can then be added to the standard contingency list used by the energy management system (EMS) for transmission security assessment.

Transmission substations are normally designed to ensure that a single fault results in at most loss of a single circuit. However, the actual substation topology, at any given moment, may differ from the designed configuration, as the topological configuration of a substation, in terms of the connectivity of the elements through the switching devices (switches and breakers), may change. Variations in substation topology can occur as a result of operator action for purposes of facility maintenance and for purposes of mitigating undesirable operating conditions such as high circuit loading or out-of-limit voltages. To a lesser extent, topological variation may also occur as a result of forced outages.

Substation topological variation may, in some instances, result in situations where the operation of the protective systems, in response to the occurrence of a fault in the network, removes two or more elements when clearing a fault. Such topologies significantly increase the risk-level of the network, as it exposes the system to an N-k contingency as a result of a single fault, with probability equivalent to that of an $N-1$ contingency. As $N-k$ contingencies are inherently more severe than $N-1$ contingencies, an $N-k$ contingency having a probability of the same order of magnitude as an $N-1$ contingency may cause a very high amount of risk. Here, the risk associated with a specific contingency is defined to be the expected value of the contingency consequence, or the summation of all possible consequences weighted by their probabilities [9].

Without automated detection, an operator may not observe an $N-k$ probability increase as a result of switching actions, and if such an increase is noticed, the magnitude of that increase may not be obvious. A graph-search algorithm and associated code were developed to detect these situations and estimate their probability. The inputs required for the algorithm include the breaker-switch status data obtained from the SCADA system. As this data is also used for EMS topology processing, it is available in most control centers.

Another cause of $N-k$ events is the failure of a breaker to open under a faulted condition. There are two causes for a breaker to fail to open: the breaker itself has problem or the protection fails to send out the command signal to the breaker. Such an event is lower in probability than that of an $N-1$ outage, as it is comprised of a fault and a protection system failure. Yet, the severity, in terms of number of outaged elements, may be extreme, and therefore the risk may be non-negligible. The graph-search algorithm developed also detects this situation.

The NERC Disturbance Analysis Working Group (DAWG) provides a database on major disturbances that have occurred in the bulk transmission systems in North America since 1984 [10]. Our analysis of this information resulted in a classification of three types among those related to protection failures: 1) inadvertent tripping, 2) protection relay fail to trip, and 3) breaker failure. A summary of the DAWG database in terms of this classification is given in Table I. If this table represents accurate statistics, then our approach addresses all the failures in category 3 (34%). Since our approach assumes only one breaker fails each time, part of the failures in category 2 (11%) will also be addressed. The total protection failures addressed should be between 34% and 45%.

Section III describes a conceptual underpinning of this work – the rare event approximation. Section IV illustrates how topological changes may place power systems at high risk. Section V describes through an example our graph-search and probability estimation algorithms. Section VI addresses algorithm scalability. Section VII provides results obtained from applying the approach to topology data obtained from the EMS of a large US utility. Section VIII concludes.

TABLE I
Summary on disturbances caused by protection system failures

Year	Category 1 Inadvertent Tripping	Category 2 Protection fails to trip	Category 3 Breaker Failure	Total No. protection malfunction
1984	4	0	1	5
1985	2	0	5	7
1986	1	1	2	4
1987	2	0	0	2
1988	6	0	0	6
1989	6	0	0	6
1990	0	2	1	3
1991	3	1	1	5
1992	1	1	2	4
1993	1	0	3	4
1994	2	0	3	5
1995	5	1	1?	7
1996	2	0	1	3
1997	1	0	2	3
1998	0	0	0	0
1999	0	1	0	1
Total	36	7	22	65
Per.	55%	11%	34%	100%

III. RARE EVENT APPROXIMATION AND EVENT TREE

In this section, the rare event approximation and the event tree are introduced. These two concepts underpin our approach for topology-driven contingency selection.

A. Rare Event Approximation

Suppose $p_1, p_2, \cdots, p_n$ are the individual probabilities of a group of independent events $E_1, E_2, \cdots, E_n$. The probability of a compound event, i.e., a combination of events $E_1, E_2, \cdots, E_n$, can always be expressed as a polynomial of $p_1, p_2, \cdots, p_n$. For example, the probability of the event $(E_1 \cap E_2) \cup E_3$ is $p_3 + p_1 p_2 - p_1 p_2 p_3$. Further suppose that $p_1, p_2, \cdots, p_n$ are all of approximately the same order of magnitude, then the order of magnitude of each product term in the polynomial will depend on how many terms are in the product. The number of terms in the product is called the probability order. Thus, the probability of $(E_1 \cap E_2) \cup E_3$ is composed of three different terms p_3 (probability order 1), $p_1 p_2$ (probability order 2), and $p_1 p_2 p_3$ (probability order 3).

The *probability order* indicates the order of magnitude of an event's probability. It originates from consideration of multiple independent events, with each event having occurrence probability close to P (i.e., between P and $10P$). For examples, one event occurs with probability order 1 (occurrence probability P), two events occur independently with probability order 2 (occurrence probability P^2), and so on. Event probability, even for dependent events, may be classified via probability order. In many decision problems, knowledge of the probability orders of the significant events is sufficient to distinguish among alternatives.

The basic idea of the rare event approximation is that, if the individual probabilities of a group of independent events are very small, the higher order terms of the polynomial can be omitted without much loss of precision [11]-[13]. In the given example, if p_1, p_2, and p_3 are very small, then the probability of $(E_1 \cap E_2) \cup E_3$ could be approximated as $p_3+p_1p_2$, or even as p_3.

The failure probability for most power system components is very small. Typical fault probability of one power system component has magnitude 10^{-6}/hour (or <1% per year) [14]. Suppose the fault probability of a line is p_1/ hour and the per-demand failure probability of a breaker is p_2 (about 10^{-3} in [4]). Obviously, faults and breaker failures are not exclusive events (although they are independent events [1]). The probability of a line fault (p_1), a breaker failure (p_2), or both is $p_1+p_2-p_1p_2$. Considering the small nature of p_1 and p_2, if the term p_1p_2 are ignored, the error is only about 10^{-9}. An implication is that, when dealing with rare events, the probability of a compound event is dominated by the lowest order terms, and thus the probability order is a reasonable measure of the event's probability. A detailed discussion of rare event systems can be found in [11].

B. Event Tree

The correct or incorrect operation of one part of protection often depends on the operation of another part. The actions of a protection system, whether correct or not, occur in a *sequence* rather than simultaneously. This characteristic makes the event tree [12][13] a suitable tool to model the protection failure scenario. Reference [4] used event tree to evaluate the probability of breaker stuck failure. Here event tree is used as a tool to describe cascading events as well as evaluate event sequence probability.

Fig. 1 shows an event tree describing the protection behavior of a power system after an initiating fault event.

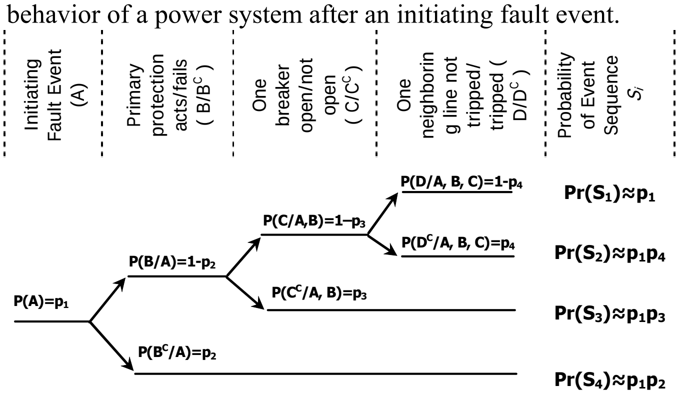

Fig.1. Event tree expansion following a line fault

[1] The event "stuck breaker" does not occur until the event "fault" occurs that generates the need for the breaker to open, and so the event "stuck breaker" is clearly dependent on the event "fault." However we consider that such an event occurs as a result of the breaker being in a stuck breaker state in advance of the fault. The occurrence of the event "stuck breaker state" and the event "fault" are therefore independent. We refer to the probability of the "stuck breaker state" as the *per-demand failure probability* of the breaker.

This tree is not expanded to a full scale. It is the useful feature of the event tree that it can be pruned according to the structure of the physical system or the probability of events. If the tree were fully expanded, there would be eight branches at the right side of the event tree rather than four. Other possible branches of the tree are cut off. For example, if there is a fault but the primary protection system (relay) fails, breaker failure has no influence on the outcome, assuming breaker failure, relay failure, and inadvertent tripping are independent events with small probability. The right side of the diagram provides the probabilities and descriptions of each event sequence. The probabilities of each node are approximated by the rare event approximation, i.e., use 1 to substitute $1-p_i$ terms. The $N-k$ events described in Section IV are branches of the event tree in Fig. 1.

IV. System topology and N−k contingnencies

Three motivating examples are provided in Fig. 2-4. Fig. 2 shows a simplified two-bus station. The three lines are connected to backup bus 2 without breakers. Normally, the three lines are connected to bus-1, bypass switches 1-3 are open, and loss of all three lines requires occurrence of a fault together with a failure of the primary protection to operate, a scenario of order-2. When bus-1 needs maintenance, breakers 1-3 are open and switches 1-3 are closed. This situation makes the substation more vulnerable than usual. Suppose line 1 has a fault. Since switches 1-3 do not have the capacity to interrupt current, the three lines have to be cleared all together, resulting in an $N-3$ contingency. Thus, the bus maintenance activity degrades an $N-3$ event from order-2 to order-1. Even under light load conditions, this can affect a considerable change in risk.

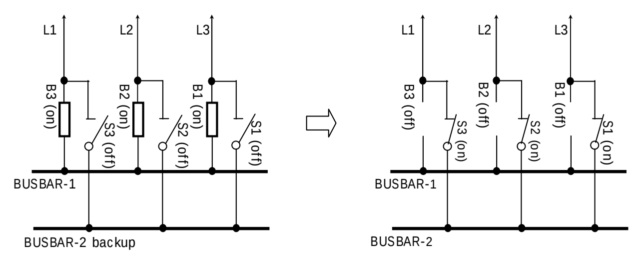

Fig. 2. $N-3$ exposure increases from prob. order 2 to prob. order 1 when performing maintenance on a double breaker-double bus configuration

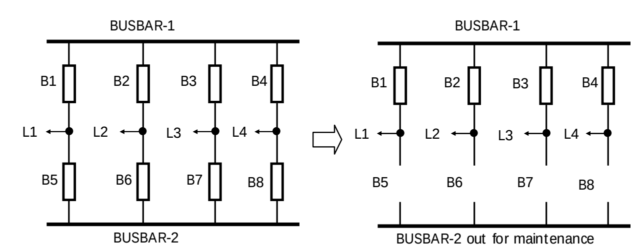

Fig. 3 Double-breaker double-bus and single-bus-single-breaker

As a second example, a substation with double breaker and double bus (DB-DB) is shown in the left of Fig. 3. This design is advantageous relative to a single-bus-single-breaker (SB-

SB) configuration because it is convenient for bus maintenance, and it is robust to high order contingencies like what would occur if a line fault were followed by failure of a primary protection system. For example, if a fault occurs on line L1, but breaker *B1* fails to open*, B2, B3* and *B4* can serve as backup to isolate the fault, limiting this order-2 scenario to an *N−1* outage. However, if one of the two buses is out of service, as shown on the right hand side of Fig. 3, a fault on line 1 followed by breaker *B1* failure to open requires that *B2, B3*, and *B4* operate as backups. Thus, an order-2 scenario results in an *N−4* event, taking the entire substation out of service.

The last example, Fig. 4, shows a ring bus substation. When B4 needs maintenance and is removed from the station, a single tripping of line 3 or 2 will cause the ring bus to be sectionalized into two and one of the remaining lines open ended.

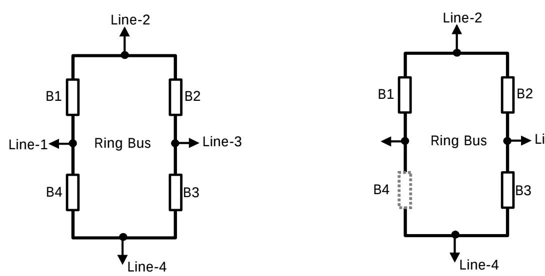

Fig. 4 Ring bus more vulnerable with the outage of one breaker

V. TOPOLOGICAL IDENTIFICATION OF N-K CONTINGENCIES

A desirable contingency selection method should be able to identify, from topology data, high risk contingencies, that is, contingencies that have relatively high probability or high consequence or both. In addition to events with probability order 1, the method proposed in this section strategically choose a group of events that have a probability less than that of order 1 but greater than or equal to that of order 2.

We assume that at most, only one breaker will suffer stuck failure, i.e., failure of two or more breakers to open when required poses negligible risk. This assumption is consistent with the rare event approximation, as long as the occurrences of different failures are independent (see footnote 1).

In this section we illustrate two high order contingencies caused by topology variation and component fault followed by one breaker failure or protection fail to trip, and we also give a concise form to calculate the probability of these events by tracing the topology of system. We use an example to explain the approach.

A. Graph representations of one-line breaker diagram

The one-line diagram in Fig. 5 shows part of a power system with bus-bar segment BS-7 out for maintenance. The components contained in each dashed circle of Fig. 5 form a functional group. A functional group does not include circuit breakers and open switches, which form the interface between two different functional groups. Generally, two functional groups have only one interfacing component. Each component is labeled with a unique identifier. Table II summarizes the Fig. 5 components. The last columns of Table II provide the failure probabilities of the components.

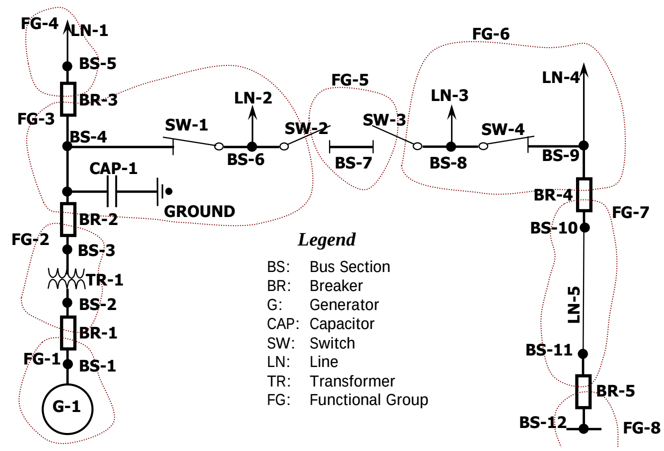

Fig. 5. One-line diagram used to illustrate functional groups and calculation of probability order

A graph *G=(V, E)* is defined by an ordered pair of finite sets *V* and *E*, where the elements in *V* are called the vertices (also called nodes or points) and the elements in *E* are called edges (also called sides or arcs) [14][15]. Each element in *E* is a subset of pairs of elements of V. For example

$$G=(V,\ E)$$
$$=\left(\{V_1,\ V_2,\ V_3\},\ \{E_1=(V_1,\ V_2),\ \mathrm{E}_2=(V_1,\ V_3),\ \mathrm{E}_3=(V_2,\ V_3)\}\right)$$

defines the triangle graph in Fig. 6 with vertices $\{V_1, V_2, V_3\}$ and edges $\{E_1=(V_1, V_2), E_2=(V_1, V_3), E_3=(V_2, V_3)\}$.

TABLE II
LIST OF COMPONENTS AND FAILURE PROBABILITIES

Name	No.	Connected Bus Sections		Status	Probability	
		from	To		Fault	Per Demand
G-1	1	BS-1	Ground	Online	P_{FT}^{1}	—
LN-1	2	BS-5	Other system	Online	P_{FT}^{2}	—
LN-2	3	BS-6	Other system	Online	P_{FT}^{3}	—
LN-3	4	BS-8	Other system	Online	P_{FT}^{4}	—
LN-4	5	BS-9	Other system	Online	P_{FT}^{5}	—
LN-5	6	BS-10	Other system	Online	P_{FT}^{6}	—
TR-1	7	BS-2	BS-3	Online	P_{FT}^{7}	—
CAP-1	8	BS-4	Ground	Online	P_{FT}^{8}	—
BR-1	9	BS-1	BS-2	Closed	0	P_{PD}^{BR-1}
BR-2	10	BS-3	BS-4	Closed	0	P_{PD}^{BR-2}
BR-3	11	BS-4	BS-5	Closed	0	P_{PD}^{BR-3}
BR-4	12	BS-9	BS-10	Closed	0	P_{PD}^{BR-4}
BR-5	13	BS-11	BS-12	Closed	0	P_{PD}^{BR-5}
SW-1	14	BS-4	BS-6	Closed	0	P_{PD}^{SW-1}
SW-2	15	BS-6	BS-7	Open	0	P_{PD}^{SW-2}
SW-3	16	BS-7	BS-8	Open	0	P_{PD}^{SW-3}
SW-4	17	BS-8	BS-9	Closed	0	P_{PD}^{SW-4}

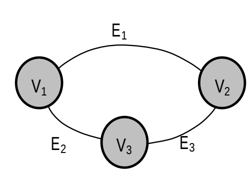

Fig. 6. A graph with three vertex and three edges

Such graphs are used to represent the topology of power system components, i.e. generators, lines, transformers, bus section, breakers, switches, and loads.

Both Fig. 7 and Fig. 8 show a graphical representation of the system shown in Fig. 5. In Fig. 7, all physical components are modeled as vertices; edges indicate connections between components and do not correspond to any physical component. In Fig. 8, both vertices and edges of the graph correspond to a physical component. The nodes are bus sections in this representation, and the edges are all other types of components. The two graphs are equivalent in that both of them completely describe the topology of the one-line breaker diagram in Fig. 5. The first representation is straight forward but apparently has more vertices and edges than the second one, which means more memory for computer. The second graph assumes that each non-bus-section physical component joins two bus sections (the two bus sections may be identical in the case of capacitors and generators.) only. This may be a problem when using it to model three-winding transformers. This problem can be solved by adding a fictitious bus section within the three-winding transformer. The functional groups are again identified with dashed circles, and each one is assigned a label FG_i. The interfacing components between each functional group are clearly visible, i.e., components *BR-1, BR-2, BR-3, SW-2, SW-3, BR-4*, and *BR-5*.

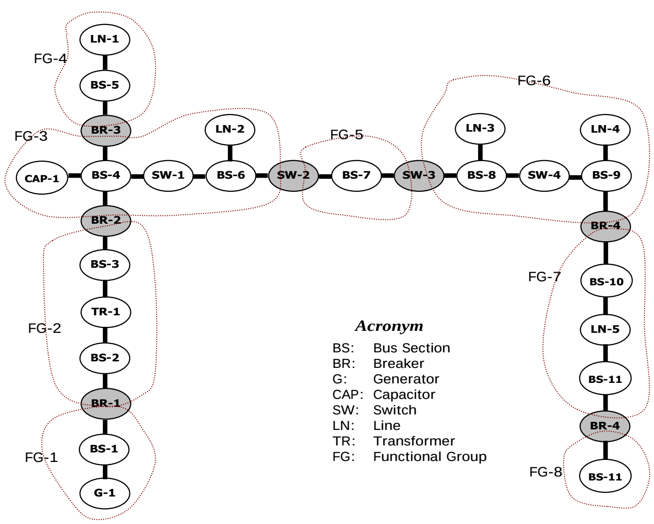

Fig. 7. Graph diagram of the configuration A

A careful inspection of the graphs Fig. 7 and Fig. 8 shows that it can be reduced to the smaller graph in Fig. 9, where all the functional groups are vertices, and interfacing components are edges. Defining (FG_i, FG_j) to be the component joining FG_i and FG_j, the new graph can be expressed by

$$G=(X, E) \tag{1}$$

where

$X=\{FG\text{-}1, FG\text{-}2, FG\text{-}3, FG\text{-}4, FG\text{-}5, FG\text{-}6, FG\text{-}7, FG\text{-}8\}$, and

$E=\{(FG\text{-}1, FG\text{-}2), (FG\text{-}2, FG\text{-}3), (FG\text{-}3, FG\text{-}4), (FG\text{-}3, FG\text{-}5), (FG\text{-}5, FG\text{-}6), (FG\text{-}6, FG\text{-}7), (FG\text{-}7, FG\text{-}8)\}=\{BR\text{-}1, BR\text{-}2, BR\text{-}3, SW\text{-}2, SW\text{-}3, BR\text{-}4, BR\text{-}5\}$

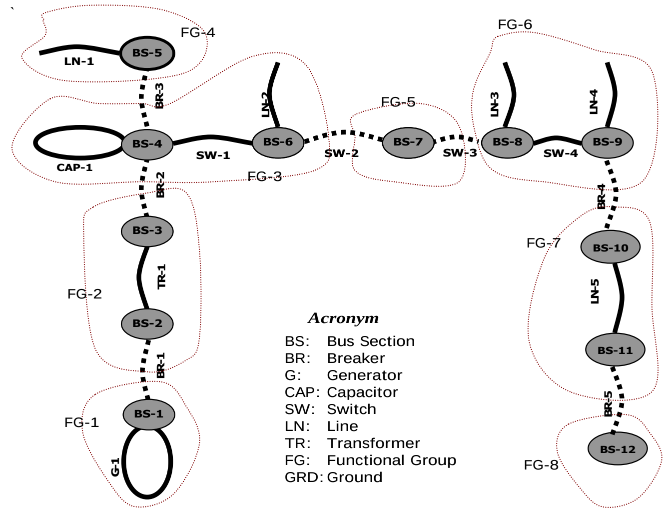

Fig. 8. Graph diagram of the configuration B

The figure below shows the graph defined by (1).

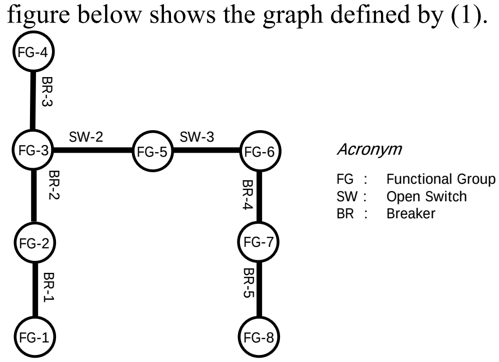

Fig. 9. Reduced functional group graph for Fig.7 and Fig. 8

Table III gives a matrix representation of the interconnections between the functional groups of Fig. 9.

TABLE III
CONNECTION RELATIONSHIP BETWEEN INTERFACING COMPONENTS AND FUNCTIONAL GROUPS (1-CONNECTED, 0-NOT CONNECTED)

-	FG-1	FG-2	FG-3	FG-4	FG-5	FG-6	FG-7	FG-8
BR-1	1	1	0	0	0	0	0	
BR-2	0	1	1	0	0	0	0	
BR-3	0	0	1	1	0	0	0	
SW-2	0	0	1	0	1	0	0	
SW-3	0	0	0	0	1	1	0	
BR-4	0	0	0	0	0	1	1	
BR-5	0	0	0	0	0	0	1	1

Table III is represented via incidence matrix [15] B, where each row of B corresponds to an interfacing component, and each column corresponds to a functional group, i.e.

$$B=\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \tag{2}$$

B. Calculating the probabilities of stuck breaker/functional group tripping

The last columns of Table II give the fault probabilities for non-switching components and the per demand failure probabilities for switching components. The fault probability of a functional group is the sum of all non-switching components in it.

$$P_{FG-i} = \sum_{component \in FG_i} P_{component}^{FT} \quad (3)$$

When two lines become part of the same functional group, the outage probability of the two lines together will be the sum of the outage probabilities for the lines individually; this means that that outage probability could approximately double.

The matrix of the per demand failure probabilities of interfacing components is denoted as

$$D = diag\left(P_{PD}^{BR-1},\ P_{PD}^{BR-2},\ P_{PD}^{BR-3},\ P_{PD}^{SW-2},\ P_{PD}^{SW-3},\ P_{PD}^{BR-4},\ P_{PD}^{BR-5}\right) \quad (4)$$

The superscripts of the elements in (4) are indices of the interfacing components in Table II.

TABLE IV
FUNCTIONAL GROUPS IDENTIFIED, THE COMPONENTS IN THEM, AND THEIR FAILURE PROBABILITIES

Funct. group	Interfacing Components (breaker or open switch)	Per Demand Failure Probabilities	Internal-Components in FG_i	Failure Probability of Functional groups P_{FGi}
FG_1	BR-1	P_{PD}^{9}	G-1, BS-1	$P_{G-1}^{FT} + P_{BS-1}^{FT}$
FG_2	BR-1, BR-2	P_{PD}^{9}, P_{PD}^{10}	TR-1, BS-2, BS-3	$P_{TR-1}^{FT} + P_{BS-2}^{FT} + P_{BS-3}^{FT}$
FG_3	BR-2, BR-3, SW-2	P_{PD}^{10}, P_{PD}^{11}, P_{PD}^{14}	CAP-1,BS-4, SW-1, BS-6, LN-2	$P_{CAP-1}^{FT} + P_{BS-4}^{FT} + P_{SW-1}^{FT} + P_{BS-6}^{FT} + P_{LN-2}^{FT}$
FG_4	BR-3	P_{PD}^{11}	BS-5, LN-1	$P_{BS-5}^{FT} + P_{LN-1}^{FT}$
FG_5	SW-2, SW-3	P_{PD}^{14}, P_{PD}^{15}	BS-7	P_{BS-7}^{FT}
FG_6	SW-3, BR-4	P_{PD}^{15}, P_{PD}^{12}	BS-8, LN-3, SW-4, LN-4, BS-9	$P_{BS-8}^{FT} + P_{LN-3}^{FT} + P_{SW-4}^{FT} + P_{LN-4}^{FT} + P_{BS-9}^{FT}$
FG_7	BR-4, BR-5	P_{PD}^{12}	BS-10, LN-5, BS-11	$P_{BS-10}^{FT} + P_{LN-5}^{FT} + P_{BS-11}^{FT}$
FG_8	BR-5	P_{PD}^{12}, P_{PD}^{13}	BS-12	P_{BS-12}^{FT}

If a component within either of the neighboring functional groups FG_i and FG_j has a fault and the breaker connecting them fails to open, all the components in the two neighboring functional groups will be removed from service. Thus, the probability that the functional group FG_i and FG_j both fail during the time interval Δt can be expressed as

$$P_{i,j} = P_{PD}^{k_{i,j}} \times \sum_{component \in FG_i \cup FG_j} P_{component}^{FT} = P_{PD}^{k_{i,j}} \times \left(P_{FG-i} + P_{FG-j}\right) \quad (5)$$

where $P_{PD}^{k_{i,j}}$ is the per demand failure probability of the interconnecting components between functional groups FG_i and FG_j and $P_{component}^{FT}$ is the failure probabilities of a component in functional groups FG_i and FG_j. Equation (5) can be expressed in matrix form as:

$$\begin{pmatrix} P_{1,2} \\ P_{2,3} \\ P_{3,4} \\ P_{3,5} \\ P_{5,6} \\ P_{6,7} \\ P_{7,8} \end{pmatrix} = \begin{pmatrix} P_{PD}^{BR-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_{PD}^{BR-2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{PD}^{BR-3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{PD}^{SW-2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{PD}^{SW-3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{PD}^{BR-4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_{PD}^{BR-5} \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \times \begin{pmatrix} P_{FG1} \\ P_{FG2} \\ P_{FG3} \\ P_{FG4} \\ P_{FG5} \\ P_{FG6} \\ P_{FG7} \\ P_{FG8} \end{pmatrix} \quad (6)$$

or

$$P = D \times B \times P_{FG} \quad (7)$$

where B is given by (2) and D is given by (4). As shown in Fig. 7, switches SW-2 and SW-3 are open, so it is not possible for these to fail to open. Therefore P_{PD}^{SW-2} and P_{PD}^{SW-3} are zero.

C. FG-Decomposition of system topology

This part addresses the identification of the functional groups and their connections in the form of the connection matrix B from topology data. Central to this process is the algorithm for decomposing the network into functional groups, referred to as *FG-decomposition*.

The topology data for each substation, as summarized in columns 1-5 of Table II, is available within the EMS. This is reasonable since almost all EMS today have a topology processing function which requires this data as input in order to create (with the state estimator) the system model. The FG-decomposition algorithm processes this data using a graph traverse technique [14] based on a breadth-first search (BFS). The complexity of standard BFS searching algorithm is o(n+e) for graph connection represented in the form of adjacency lists and o(n^2) for graph connection represented in the form of adjacency matrix, where n is the number of vortices, and e is the number of edges. The use of a first-in-first-out (FIFO) data list is critical for the BFS. Starting from an unvisited vertex (bus section), our program uses the BFS to visit and list all the internal and boundary components of the FG that contains the starting bus section. This process repeats for all unvisited vertices connected until there are no more such vertices. This algorithm, denoted as Algorithm 1, is summarized below, where FG stands for functional group and BS stands for bus section.

Algorithm 1 FG-Decomposition

1) label all the components as 'unvisited'
2) indexFG ⟵ 0, indexBS ⟵ 0: initialize indices
3) loop-0; loop through all the unvisited bus sections with indexBS be the counter
4) indexBS ⟵ indexBS+1: Note the starting component is always a bus section component
5) if the bus section indexBS is visited and it is not the last bus section in the power system considered, repeat step 4
6) endif
7) if the bus section indexBS is visited and it is the last the bus section in the power system considered, then end
8) endif
9) FIFO ⟵ Null: clear FIFO link list
10) indexFG=indexFG+1, $FG_{indexFG}$ *⟵ Null*
11) add $BS_{indexBS}$ *to FIFO*
12) add $BS_{indexBS}$ *to* $FG_{indexFG}$
13) label $BS_{indexBS}$ *as visited*
14) loop-1:While FIFO!=null,
15) u ⟵ pop one element from FIFO
16) loop-2: for each w that is immediate neighbor of u
17) if w is non-interfacing component, then
18) if w is unvisited, then
19) add w to FIFO
20) add w to $FG_{indexFG}$
21) label w as visited
22) endif
23) else if w is breaker or open switches
24) if w's starting functional group is null then
25) w's starting functional group ⟵ $FG_{indexFG}$;
26) else if w's ending functional group is null then
27) w's ending functional group ⟵ $FG_{indexFG}$

```
                endif
            endif
        end of loop-2
    end of loop-1
end of loop-0
end of FG-decomposition
```

Suppose the time spent on identifying functional group i is t_i, then the total time for a full FG-decomposition is $\sum t_i$. Suppose N is the total number of functional groups in the system, the time for

Algorithm 1 is bounded by $N \times max\{t_i\}$. where $max\{t_{ave}\}$ is always small since the design of a power system stipulates the size of a functional group cannot be too large.

D. Decompose in updating mode

Operationally, the full network FG-decomposition of algorithm 1 need not be run frequently. After an initial run, the identified functional groups and associated contingency probabilities can be simply updated, where the updating is triggered only when a switching operation takes place. There are only four basic switching operations that change the connectivity of a power system. They are *opening a breaker, closing a breaker, opening a switch,* and *closing a switch.*

In updating for one of these four switching operations, opening/closing a breaker changes only the stuck breaker probabilities of (6) but not the identity or composition of the functional groups. That is, the status change of a breaker does not change the need to distinguish between the groups of components on either side of the breaker as groups that will function or fail together. For example, independent of whether the breaker *BR-2* in Fig. 7 is open or closed, functional groups *FG-2* and *FG-3* must be identified.

In contrast to the case of opening/closing of breakers, opening/closing a switch does change the identity of the functional groups. Closing a switch merges the two functional groups it connects into one functional group. For example, if switch *SW-2* in Fig. 7 is closed, then the two functional groups *FG-3* and *FG-5* are merged into one functional group since any faulted component within either of these groups removes all components within both. On the other hand, opening a switch splits a single functional group into two.

The algorithm to decompose power system in the update mode is summarized as follows. In the algorithm below, SO stands for switching operation.

Algorithm 2 DG-decomposition in update mode

```
SO ← Secure one switching operation from EMS/SCADA
if SO is to turn off/on breaker i, then
        breaker i's status ← OFF/ON
else if SO is to turn off switch i, then
        switch i's status ← OFF
        rightBS ← the right bus section switch is connected with
        leftBS ← the left bus section switch is connected with
        rightFG ← starting from rightBS, find the functional
                  group that contains rightBS.
        leftFG ← starting from leftBS, find the functional group
                  that contains leftBS. Denote
        switch i's starting functional group ← rightFG
        switch i's ending functional group ← leftFG
else if SO is to turn on switch i, then
        switch is tatus ← ON
        newFG ← switch i's first functional group + switch i's
                  second functional group
        switch i's functional group ← newFG
endif
end of update for SO
```

E. Topological special cases

Two topological special cases can arise. The two cases are described below in terms of breakers as they would more likely appear that way, but it is conceivable that they could arise in terms of open switches.

E.1 Two interfacing components in parallel

In some cases, in order to avoid isolation during breaker maintenance, two breakers may be paralleled as shown in Fig. 10.

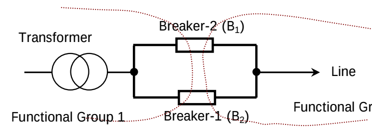

Fig. 10. Two breakers acting as backup of each other (Excerpt from the IEEE RTS-96 [16])

In terms of graph representation, this situation is equivalent to the case where there is more than one edge between two vertices. This type of graph is called *p-graph*, where *p* is the maximum number of edges between two vertices [15]. We add one more row in the *B*-matrix to show that another breaker is joining the two functional groups. The stuck breaker probability may be computed by viewing the two breakers as one so that the aggregated *per demand failure probability* is:

$$P(B_1 \text{ } stuck \text{ OR } B_2 \text{ } stuck) = P(B_1 \text{ } stuck) + P(B_2 \text{ } stuck) - P(B_1 \text{ } stuck \text{ AND } B_2 \text{ } stuck) \quad (8)$$

Another way is to model them as they are, i.e. to treat the stuck breaker trip due to breaker-1 or breaker-2 as different contingencies. In this case, the form of (6) is not changed and we do not need to derive the failure probability for the lumped breaker.

E.2 Interfacing and non-interfacing components in parallel

Fig. 11 illustrates this case. There is no apparent design rationale for it, but it could arise as a result of switching anomalies.

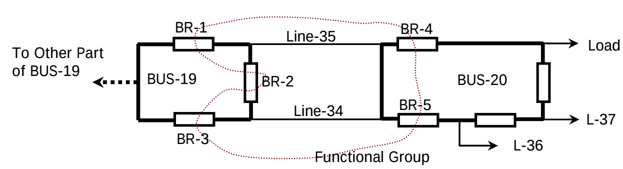

Fig. 11. Breaker BR-2 connects one rather than two functional groups (Excerpt from IEEE RTS-96 [16])

Here, the breaker *BR-2* does not actually separate two FGs. In terms of graph representation, this situation is equivalent to the case where one edge starts from one vertex and ends at the same vertex. This edge is called a *ring* in graph theory [15]. The corresponding row in the *B*-matrix as defined in (2) has one element 1 and all other elements 0. The stuck breaker contingency that corresponds to breaker *BR-2* has no influence on the FG identities or the contingency probabilities independent of whether it opens or closes.

VI. Number of Events

It is important in security assessment to be aware of the number of contingencies to be assessed. In our approach, this would equal the number of contingencies from faults resulting in single functional group trippings plus the number of contingencies from faults followed by stuck-breakers. The number of single functional group tripping contingencies is just the number of functional groups, which should be proportional to the system size. The number of stuck-breaker contingencies is the number of breakers acting as interfacing components, which should also be proportional to the system size. So the total number of contingencies is just proportional to the system size of power system studied.

VII. Test Results For FG-Decomposition

The algorithms were tested using the IEEE-RTS96 [13] and a system obtained from the EMS of a large US utility company. The IEEE RTS96 was used because it was small and was therefore convenient for debugging, and also because it was the only well-known test system we know that has full substation topology and component reliability data.

Results for this analysis are summarized in Tables V to VIII. The count k includes only lines, transformers, and generators. The functional groups identified by our algorithms may be easily verified from inspection of the topology data given in [16]. We removed a few redundant breakers (see the discussion part of [16].). In order to present a more practical scenario, a random sampling was done for all the generators, which resulted in the shut-down of G21, G23, G26, and G27. Reference [16] provides exhaustive data on the reliabilities of the components with the exception of the per-demand failure probability of breakers, which come from [4]. The formulas (3) and (6) are used to obtain the probabilities.

Table IX summarizes components in the utility system that were tested

One problem encountered in using the EMS data is that the switch data file does not distinguish between switches and breakers. After several discussions with a utility engineer, we decided to use several heuristic rules to distinguish between them. However, this classification is based on experience and may occasionally cause an incorrect judgment. In the long run, the EMS database must provide necessary fields to enable identification of breakers from switches.

TABLE V
SUMMARY OF FUNCTIONAL GROUP CONTINGENCIES FOR IEEE RTS96

k	0	1	2
No.	50	63	4

TABLE VI
SUMMARY OF FUNCTIONAL GROUP CONTINGENCY PROBABILITIES FOR IEEE RTS96

Prob/Hour	10^{-5}~10^{-4}	10^{-6}~10^{-5}
No.	32	85

TABLE VII
SUMMARY OF FAULT/BREAKER FAILURE CONTINGENCIES FOR IEEE RTS96

k	0	1	2	3
No.	24	90	50	4

TABLE VIII
SUMMARY OF FAULT/BREAKER FAILURE CONTINGENCY PROBABILITIES FOR IEEE RTS96

Prob/hour	10^{-5}~10^{-6}	10^{-7}~10^{-6}
No.	94	74

Table X summarizes contingencies caused by a single fault followed by proper protection action, and Table XI summarizes contingencies caused by a single fault followed by one stuck breaker. In both of these tables, the first row in gives the number of components lost in the contingency; the second row indicates the number of such contingencies identified. The count k includes only lines, transformers, generators, and shunts. Loads, switches, breakers and bus sections are not included in this count.

Since FG-decomposition is exclusive, the total number of components involved in those contingencies is $\sum(k \times N_k)$=3237 in Table X, which is just the total number of lines, transformer, generators and shunts in Table IX. Only 2022 (62%) out of 3237 components are protected alone by isolating breakers. So there are a considerable number of contingencies tripping more than two components. One single fault event, from Table X, outages 11 components, and one fault-breaker failure event outages 17 components. These contingencies are probably affected by the uncertainty in the data in distinguishing switches from breakers (so that some breakers are mistakenly classified as closed switches), as was mentioned above. In addition, many multi-section radial circuits, which are equivalent to a branch, are protected only by two terminal breakers. The program counts them as an N-k contingency, where k is the number of segments of the radial branches.

TABLE IX
NUMBER OF COMPONENTS IN THE UTILITY SYSTEM

Type	Bus	Line	Xfmr	Gen	Shunt	Load	Switch/Breaker
No.	1549	1830	697	353	357	1506	10653

TABLE X
SUMMARY OF FUNCTIONAL GROUP CONTINGENCIES FOR UTILITY SYSTEM

k	1	2	3	4	5	6	7	8	9	10	11
N_k	2022	468	49	14	5	3	2	1	0	0	1

TABLE XI
SUMMARY OF FAULT/BREAKER FAILURE CONTINGENCIES FOR UTILITY SYSTEM

k	1	2	3	4	5	6	7	9	10	11	12	13	14	15	17
N_k	3011	1248	356	134	63	31	23	0	1	1	7	1	0	0	1

Many contingencies involving stuck breaker trip only one component. This is mainly due to the fact that many substations use the redundant configuration such as the breaker-and-half connection. It is also in part due to the fact that we only count components that are completely disconnected. If a branch is open-ended, the program still treats it as part of the system even though it bears no load.

A full run of the program takes less than one second for a snapshot of the topology of the 1549 bus system. The computer used was a common Dell PC with Intel Pentium II processor (400MHz) and 384MB of RAM. This computation time does not include the time spent in reading the input data files into memory and processing the system topology into link lists, which typically takes a much longer time. This time cost is not counted because the algorithm is intended to be built into EMS software, so that we can assume online topology information would be pre-processed into adjacency lists for purposes such as state estimation. Power system topology changes are typically localized and incremental, involving only a few components at a time. That means if we use the updating algorithm, the processing would require little time. However, as Table XII indicates, even for a system as large as 10,000 buses, it takes only a few seconds.

TABLE XII
SEARCHING TIME TO IDENTIFY CONTINGENCIES

System	IEEE RTS 96	Large System	Projected Systems	
No. of Buses	24	1,549	5,000	10,000
Time/Sec.	0.01	0.63	2.08	4.17

Although the update algorithm changes the contingency list in an incremental way, the cumulative effect may be significant. It is important that every topological change leading to a contingency list change must be detected in a timely fashion, especially those that result in high probability *N-k* contingencies. These contingencies tend to have a higher severity than standard N-1 contingencies, and given they have close to or same probability order, their risk can be quite high.

VIII. DISCUSSION AND CONCLUSIONS

The selection of higher-order contingencies for on-line security analysis is investigated. The proposed approach systematically identifies two failure modes of protection related contingencies and the probabilities associated with them. The selection criteria are based on rare event approximation and event tree. As a result, the total number of all possible contingencies is limited to a number linearly proportional to the scale of the system.

The proposed approach is clear and simple in nature; yet it provides an efficient contingency prescreen to capture most high order contingencies related to protection malfunction. After this prescreen, severity screening techniques can be applied.

The contingencies identified change with the topology of system. Therefore a continuous tracing of power system configuration is required. Generally, the EMS of power system has the function of state estimation, which includes a topology processor. Thus the topology information is fully accessible and our approach requires no additional information beyond that. We only need to do standard graph search [14] to identify the connection matrix B in the algorithm.

The approach provides a systematic and rational way of identifying high risk multiple component outages to augment standard contingency lists used in control centers today. Adopting this approach within EMS requires no additional data beyond what is typically required already yet offers to significantly extend the ability of power system operators to identify high risk contingencies and prepare for them.

IX. ACKNOWLEDGMENT

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